Exploring technicolor from QCD

Yasumichi Aoki [Koboyashi-Maskawa institute, Nagoya University]

for the LatKMI collaboration

- RBRC workshop: New Horizons for Lattice Gauge Theory Computations -

May 16, 2012

名古屋大学

LatKMI collaboration

- YA, T.Aoyama, M.Kurachi, T.Maskawa, K.Nagai, H.Ohki,






K.Yamawaki, T.Yamazaki

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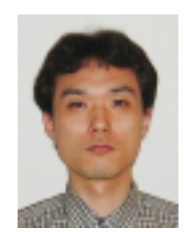


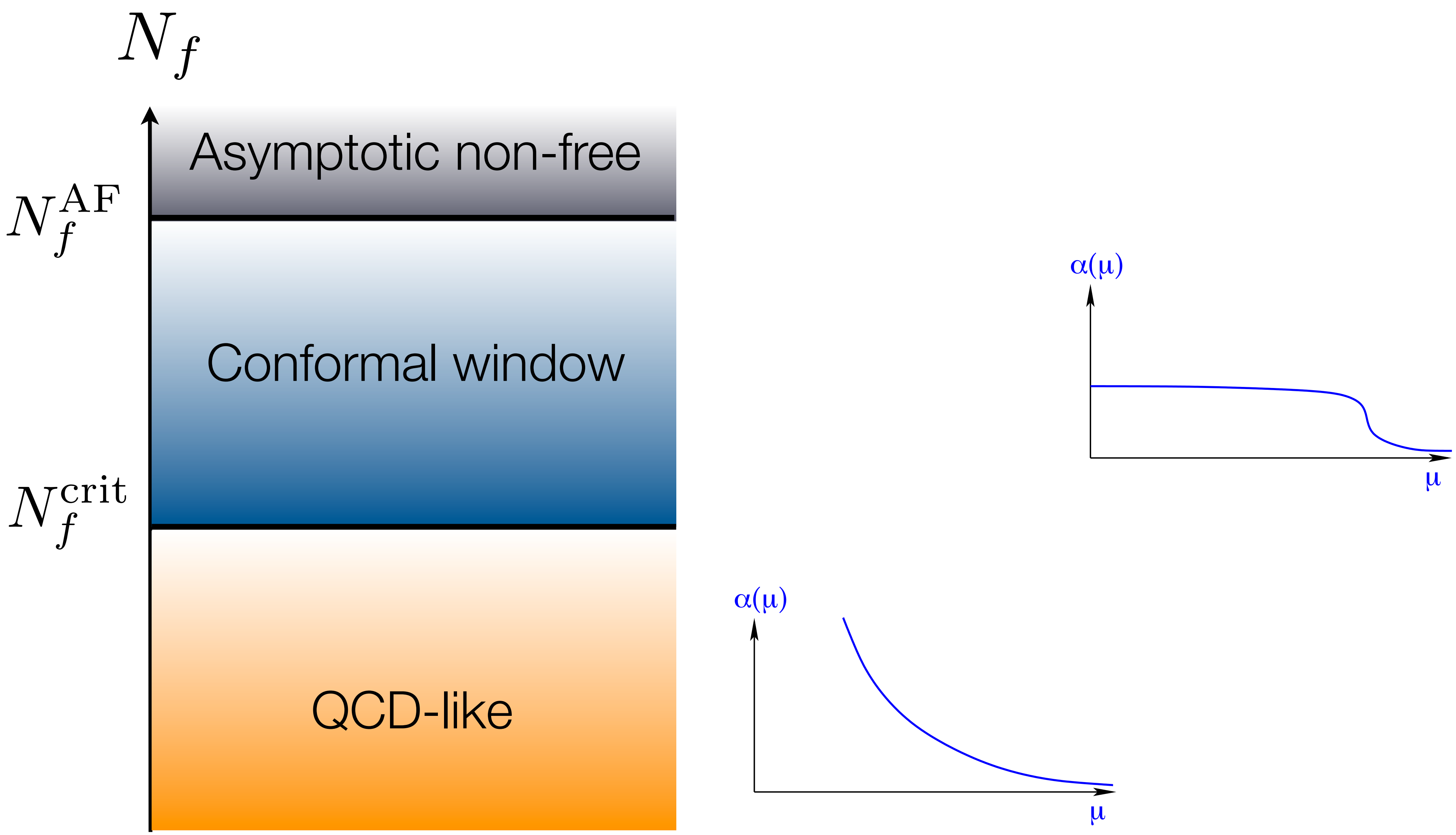
KMI

- A.Shibata

KEK

conformal window and walking coupling

- non-Abelian gauge theory with N_f massless fermions -



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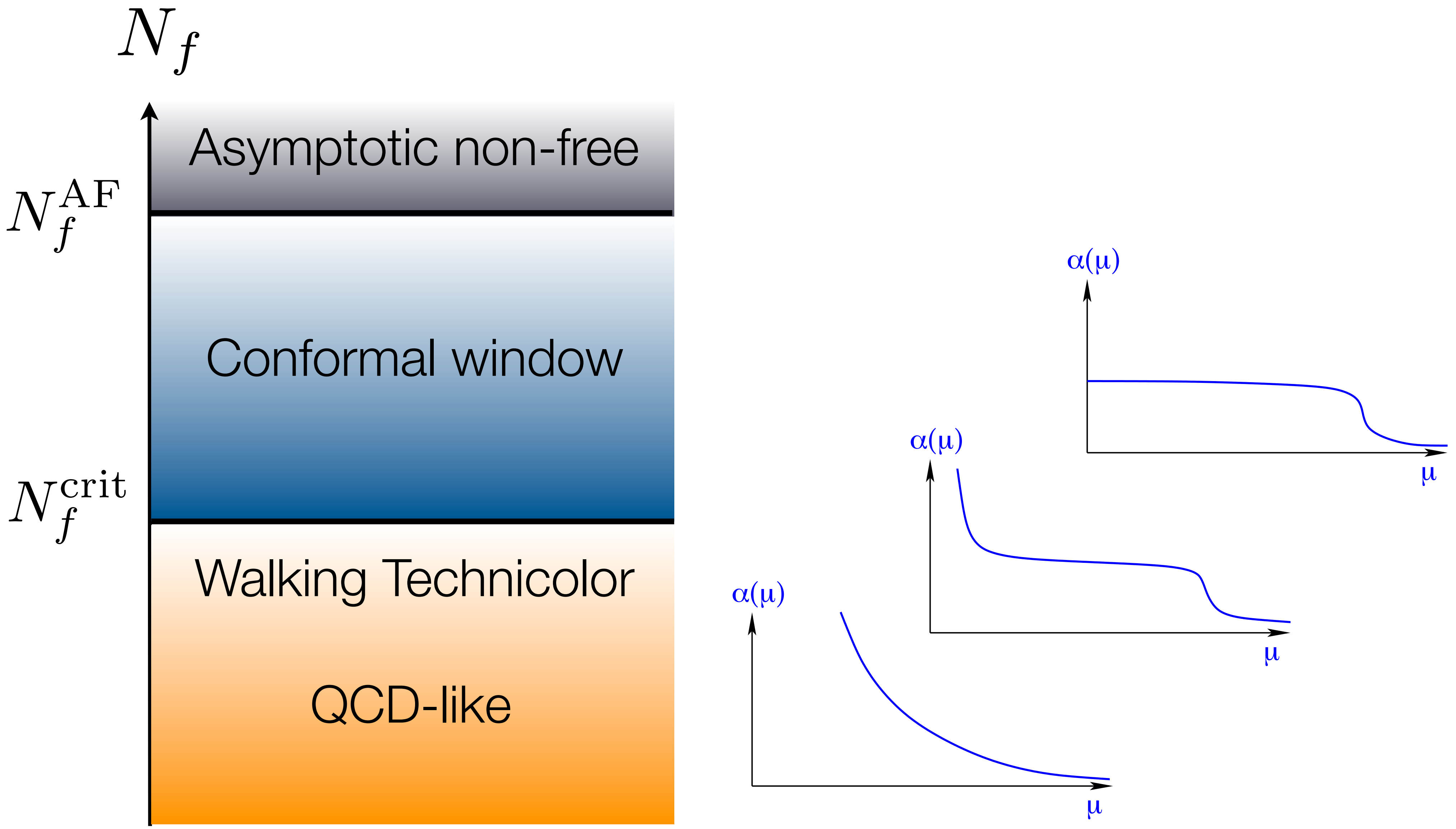

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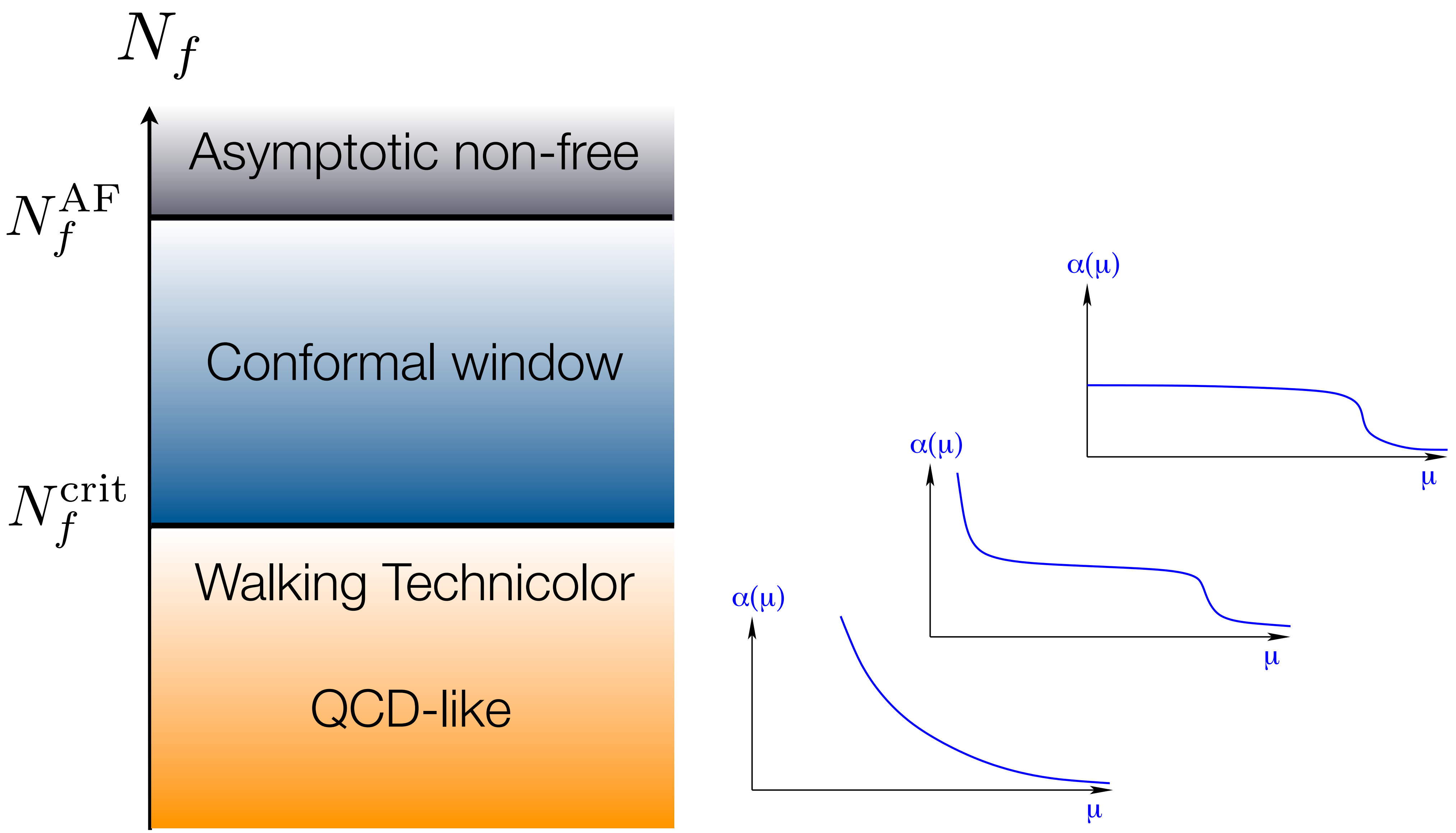

- Walking Techinicolor could be realized just below the conformal window

conformal window and walking coupling

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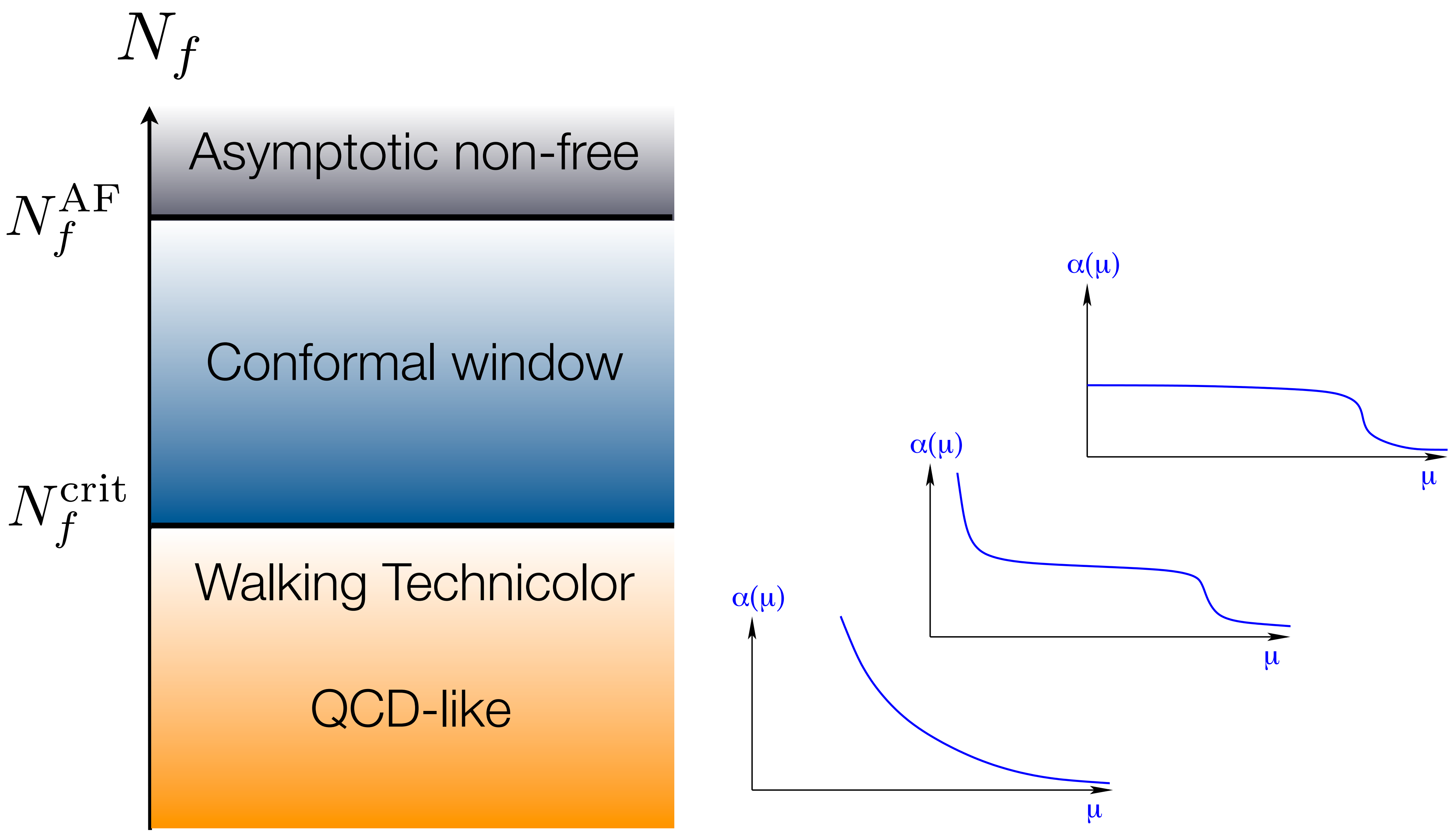
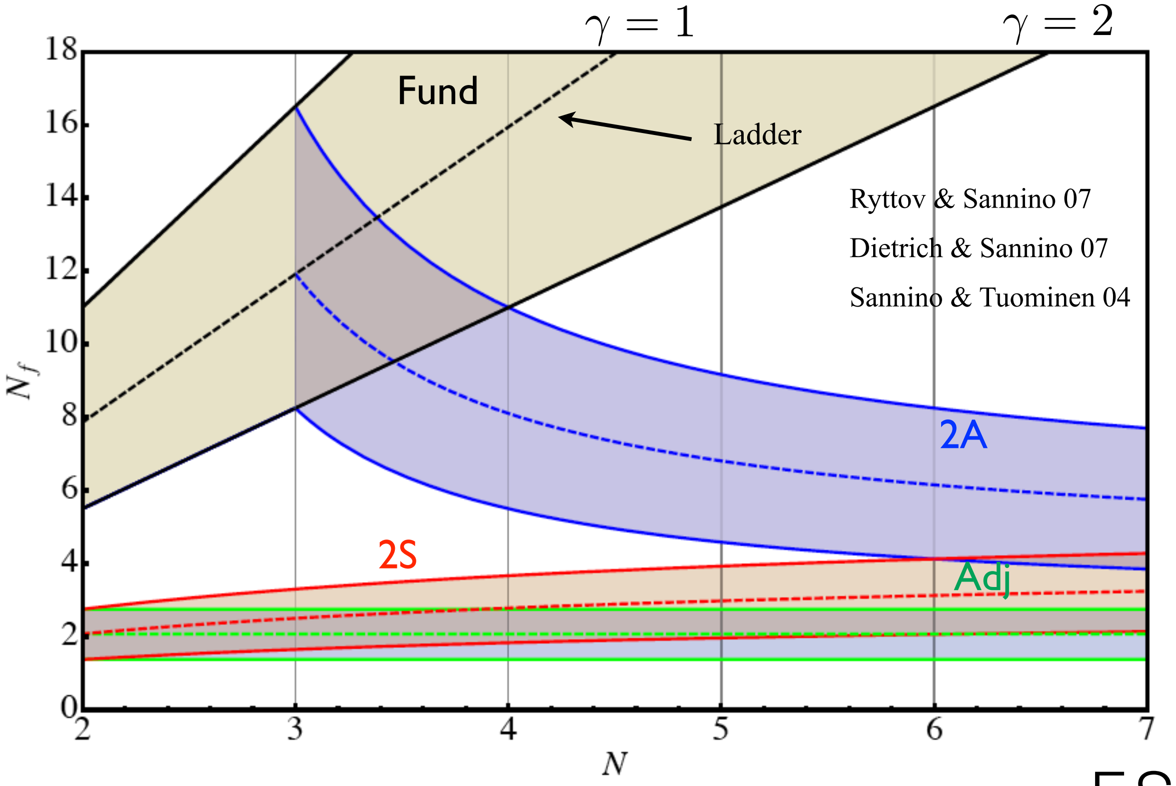


- Walking Techinicolor could be realized just below the conformal window
- crucial information: N_f^{crit} & mass anomalous dimension around N_f^{crit}

models being studied:

- SU(3)
    - fundamental: Nf=6, 8, 10, 12, 16
    - sextet: Nf=2
- SU(2)
    - adjoint: Nf=2
    - fundamental: Nf=8
- SU(4)
    - decuplet: Nf=2

SU(N) Phase Diagram



F.Sanino

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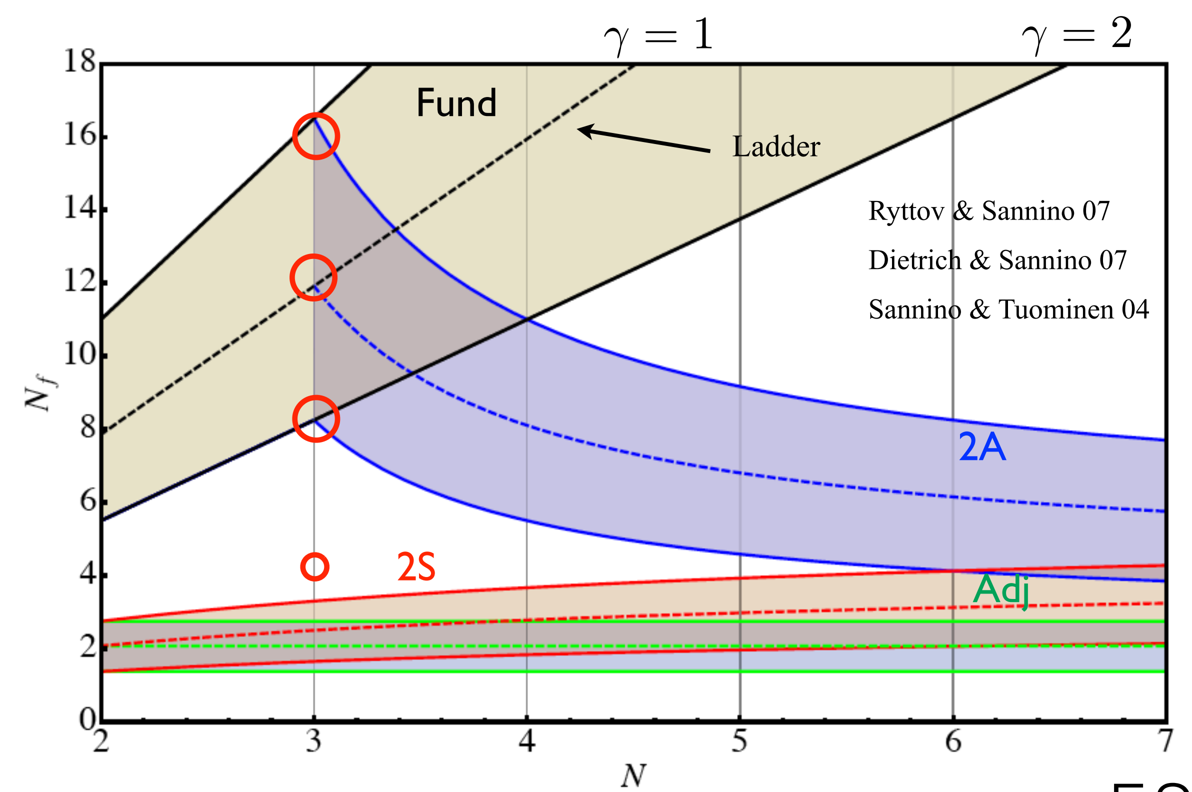

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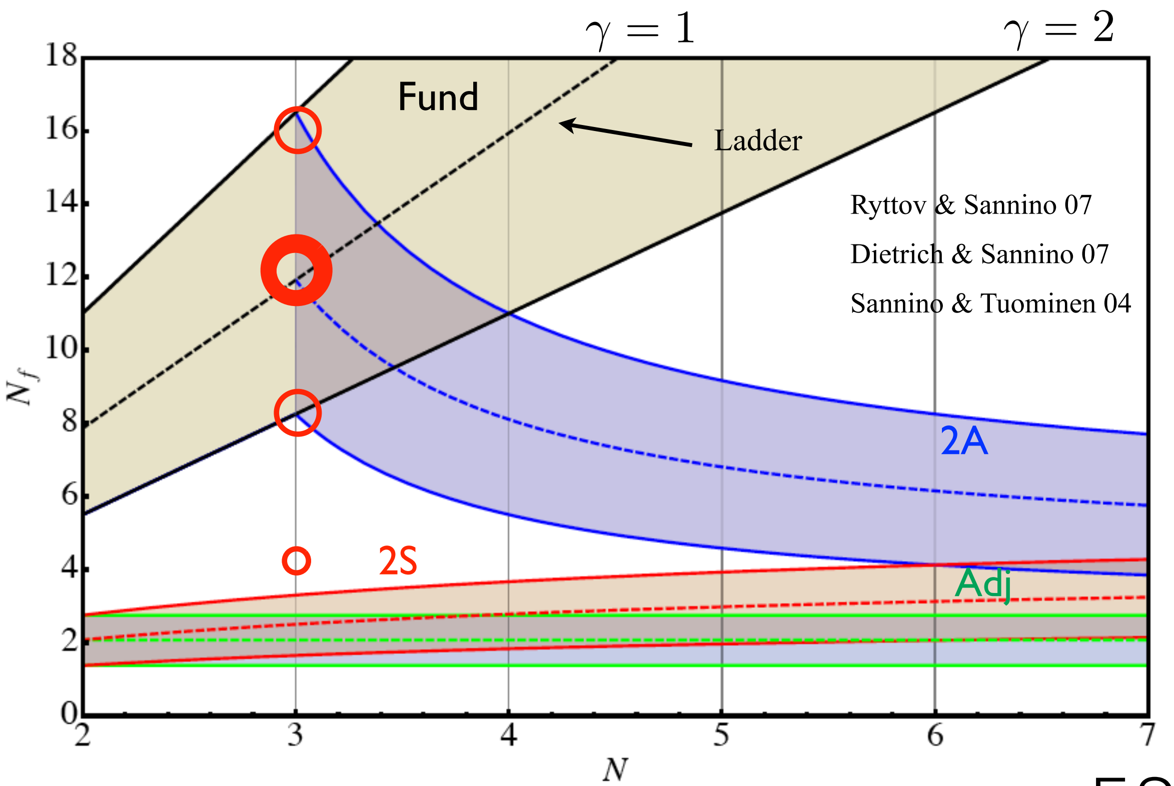

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SU(3) + N_f=12 [fundamental]

Hadron spectrum: m_f-response in mass deformed theory

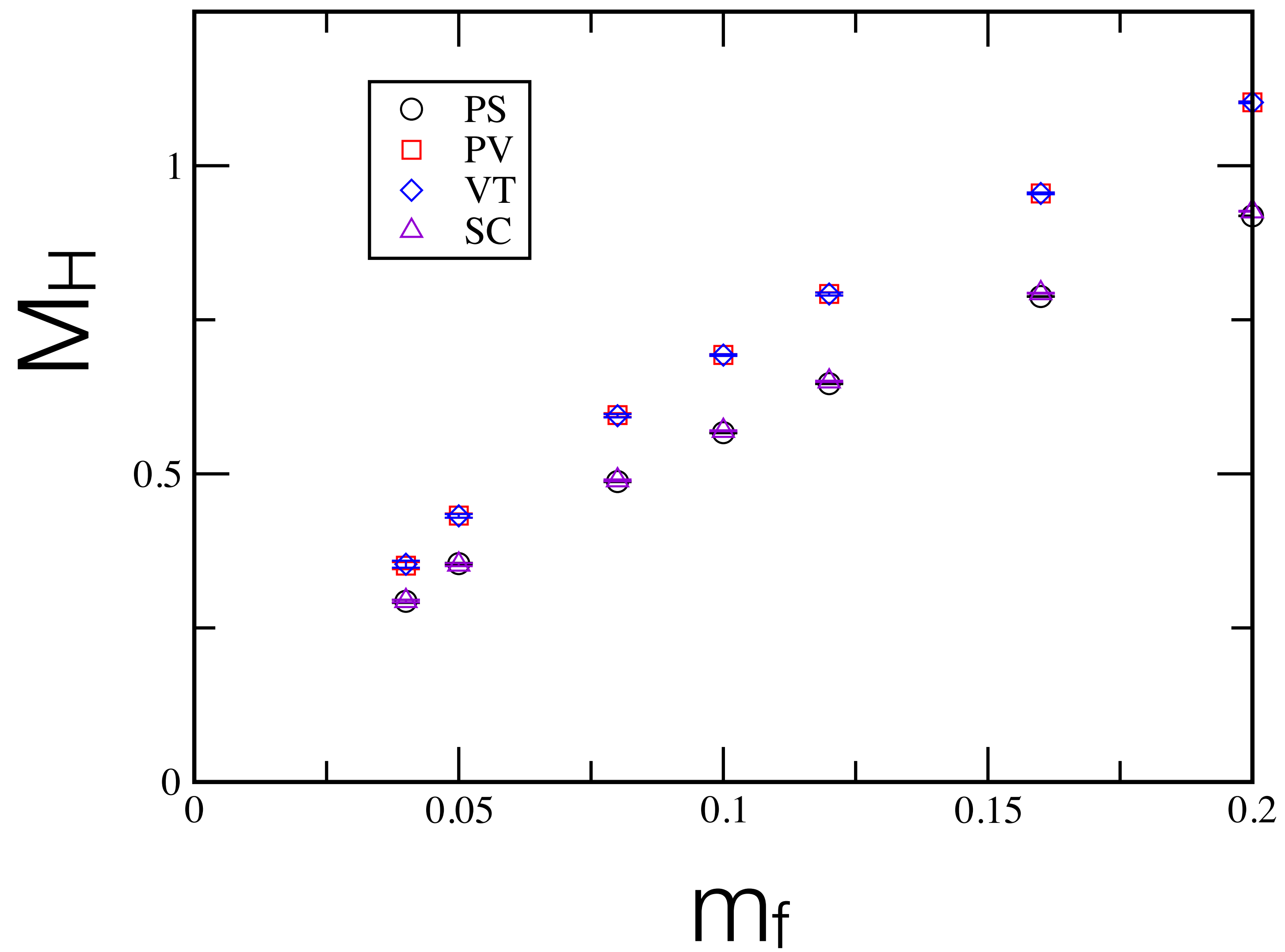
- IR conformal phase:
  - coupling runs below $\mu=m_f$: like $n_f=0$ QCD with $\Lambda_{QCD}\sim m_f$
  - multi particle / glueball state : $M_H \propto m_f^{1/(1+\gamma_m^*)}$; $F_\pi \propto m_f^{1/(1+\gamma_m^*)}$
- SχSB phase:
  - ChPT (but, large N_f, small F $\Leftrightarrow$ real QCD)
    - hard to get to the chiral regime
  - at leading: $M_\pi^2 \propto m_f$, ; $F_\pi = F + c\, m_f$
  - so far no chiral logs are observed → polynomial in m_f

Simulation

- N_f=12 HISQ (Highly Improved Staggered Quarks)
- tree level Symanzik gauge
- $\beta=6/g^2=3.7$, $V=L^3 \times T$: $L/T=3/4$; $L=18, 24, 30$, $0.04 \leqq m_f \leqq 0.2$
- $\beta=6/g^2=4.0$, $V=L^3 \times T$: $L/T=3/4$; $L=18, 24, 30$, $0.05 \leqq m_f \leqq 0.24$
- N_f=4 HISQ for the reference of SχSB for comparison
- using MILC code v7 with some modifications

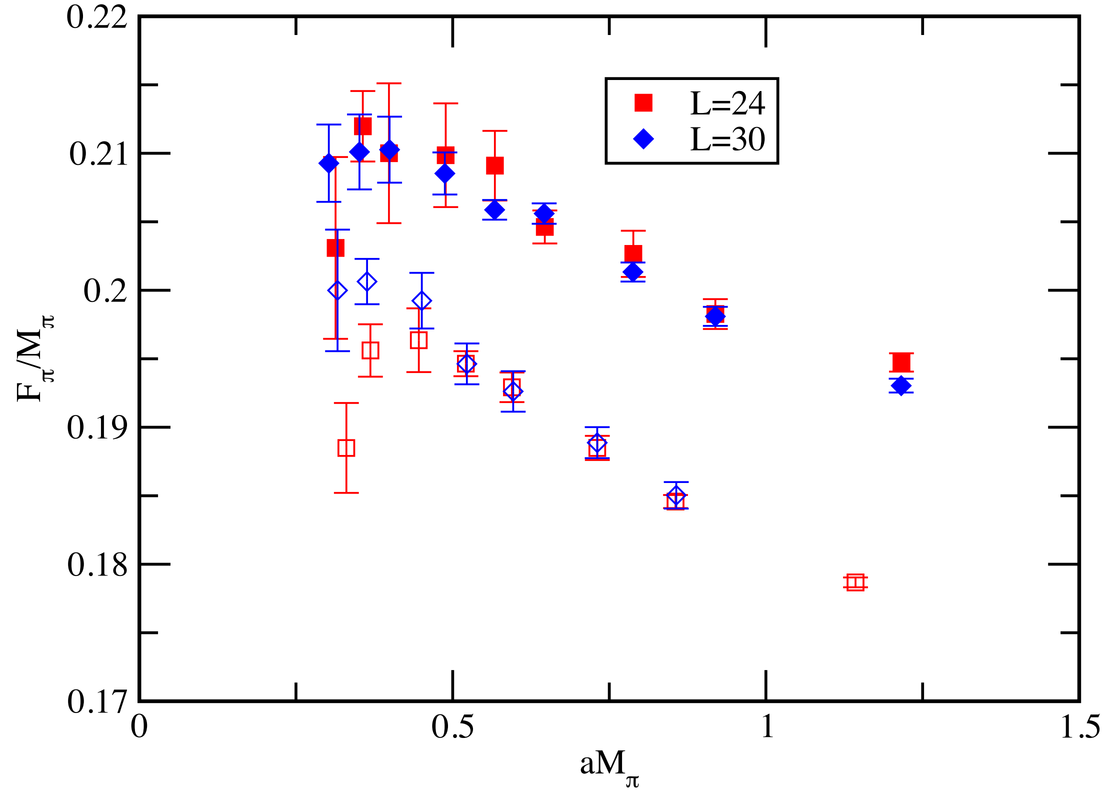
staggered flavor symmetry for N_f=12 HISQ

- comparing mesonic mass with local PS and V operators for β=3.7



a crude analysis: F_π/M_π vs M_π

N_f=12: HISQ



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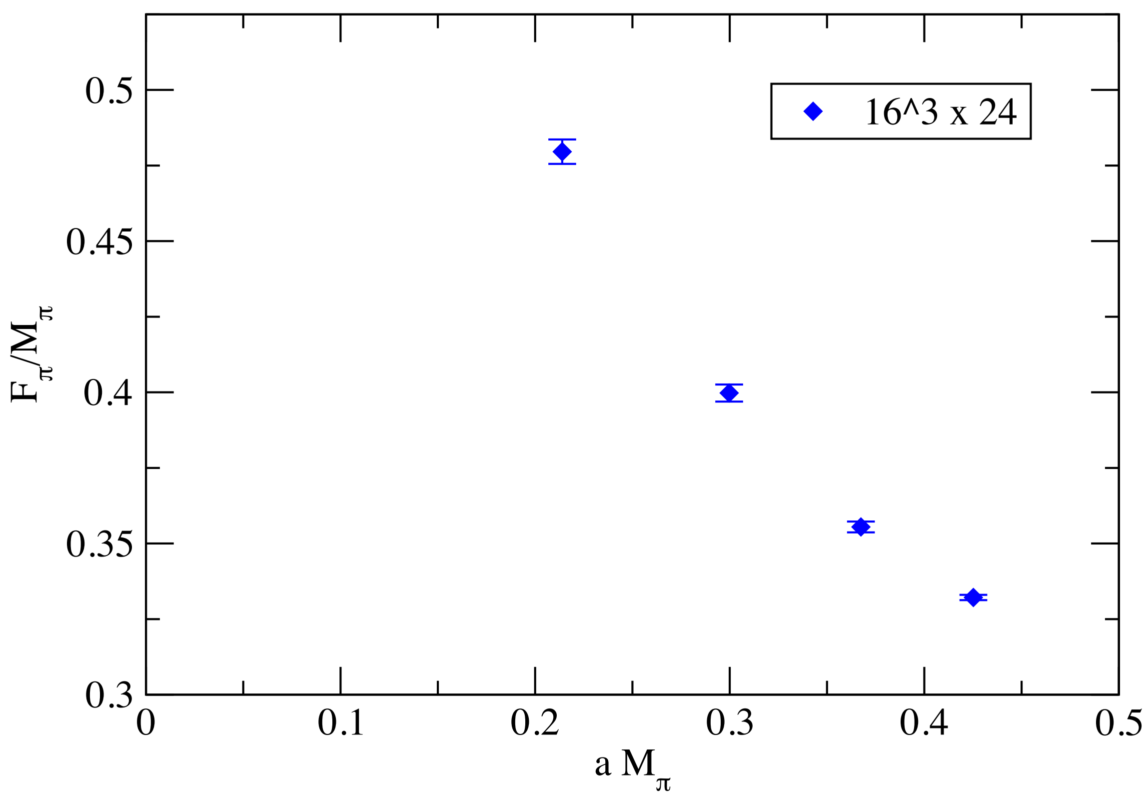

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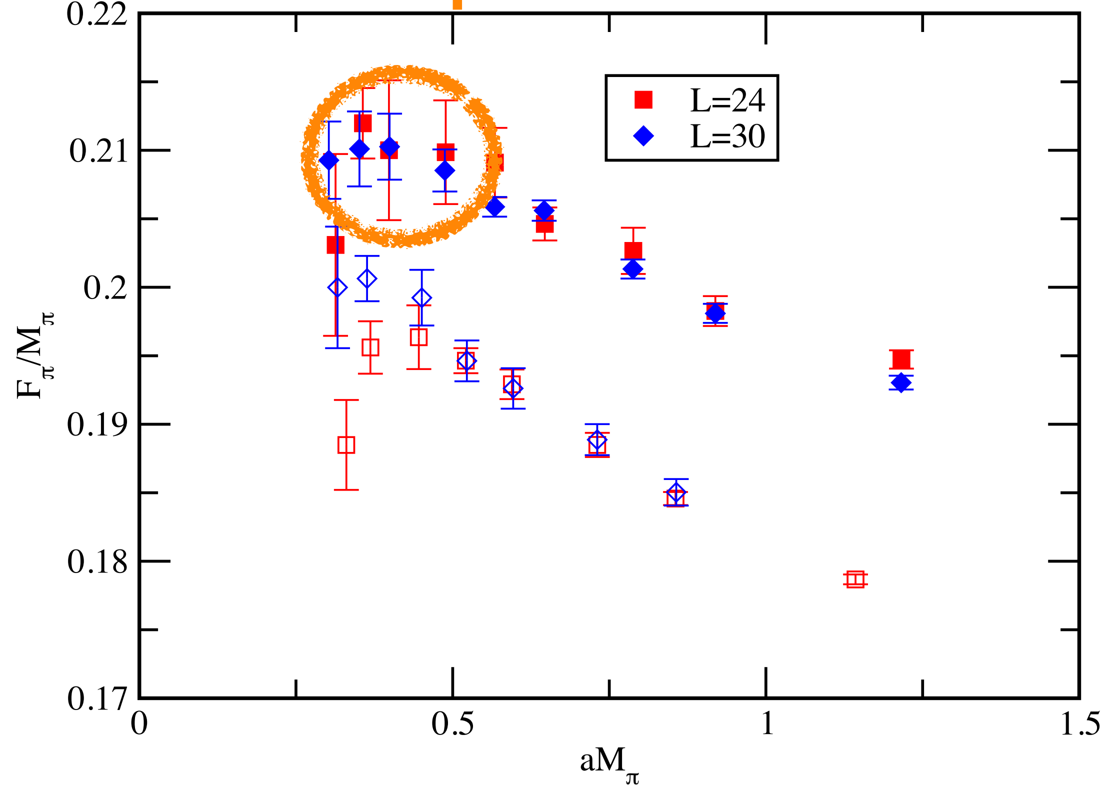
flat: β=3.7



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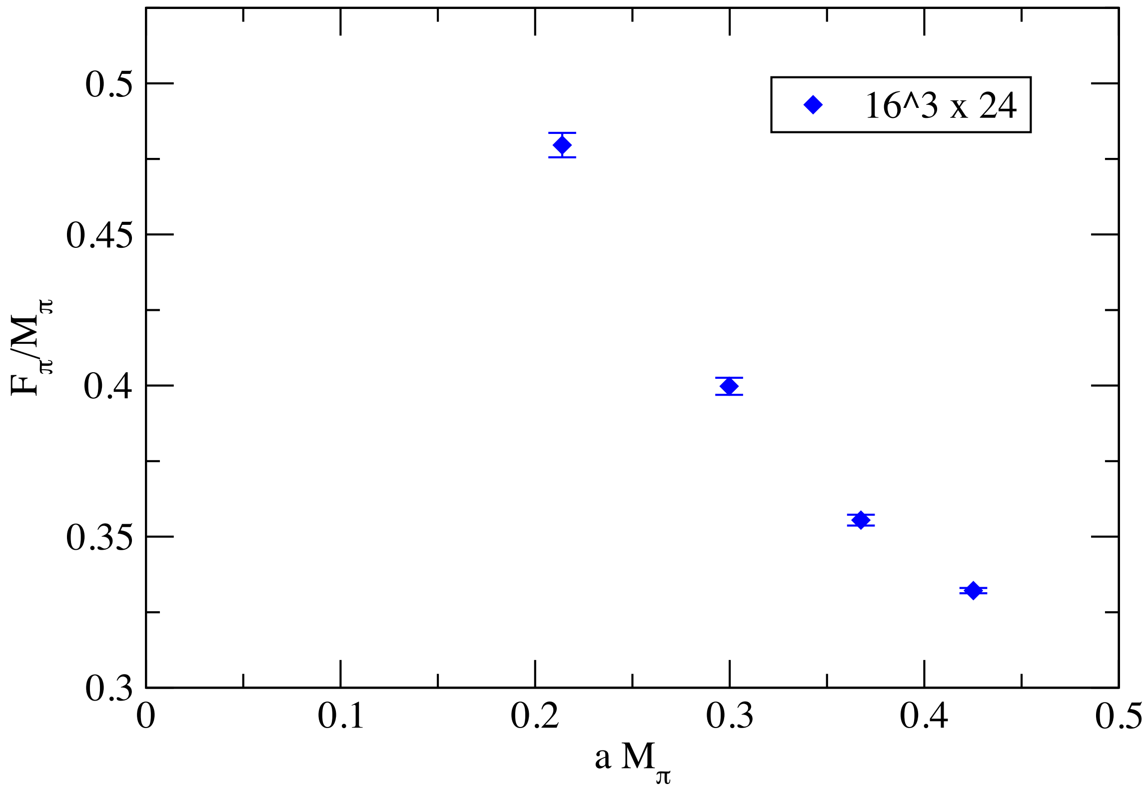

- β=3.7: small mass: consistent with hyper-scaling

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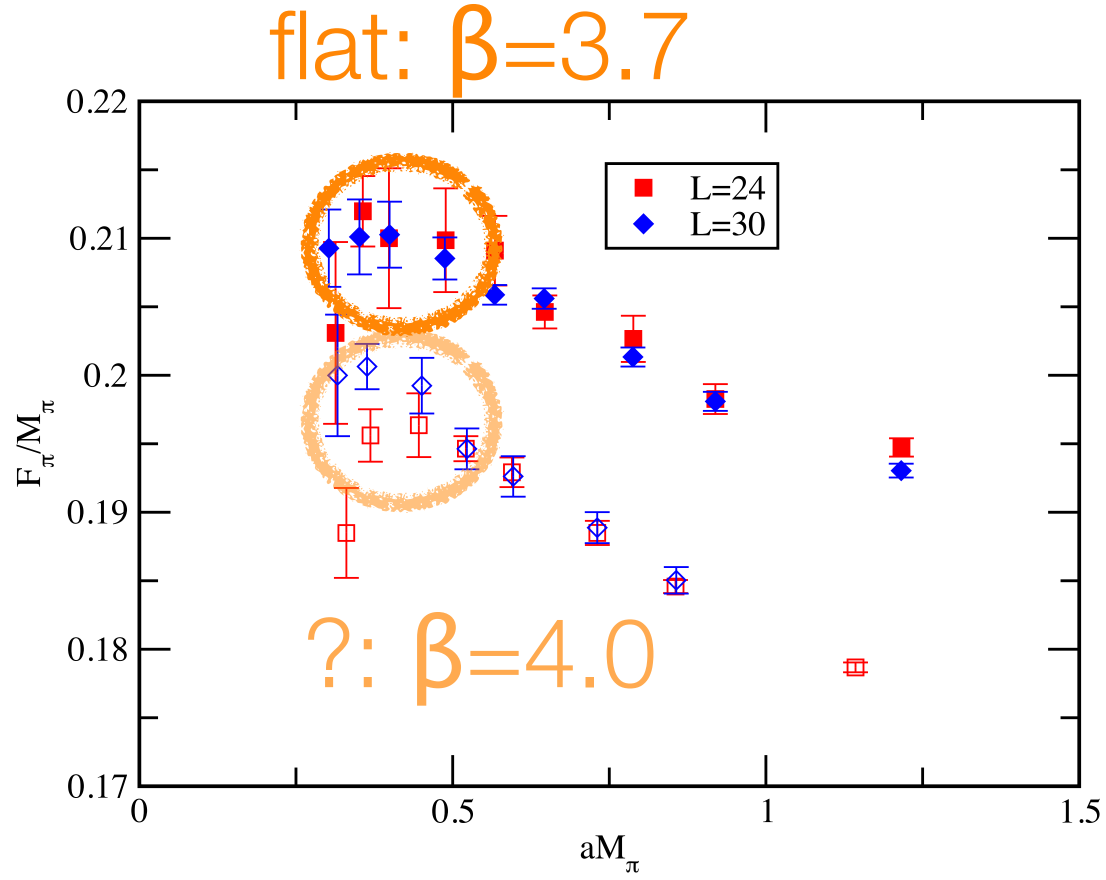

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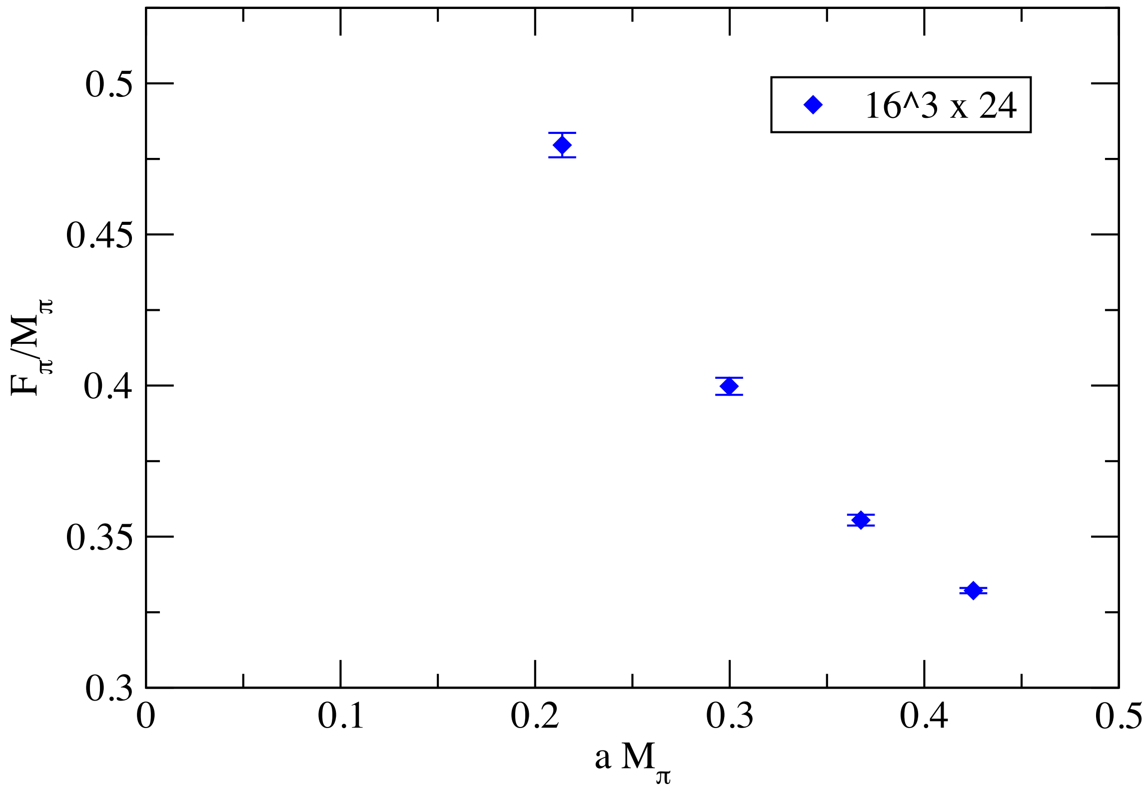
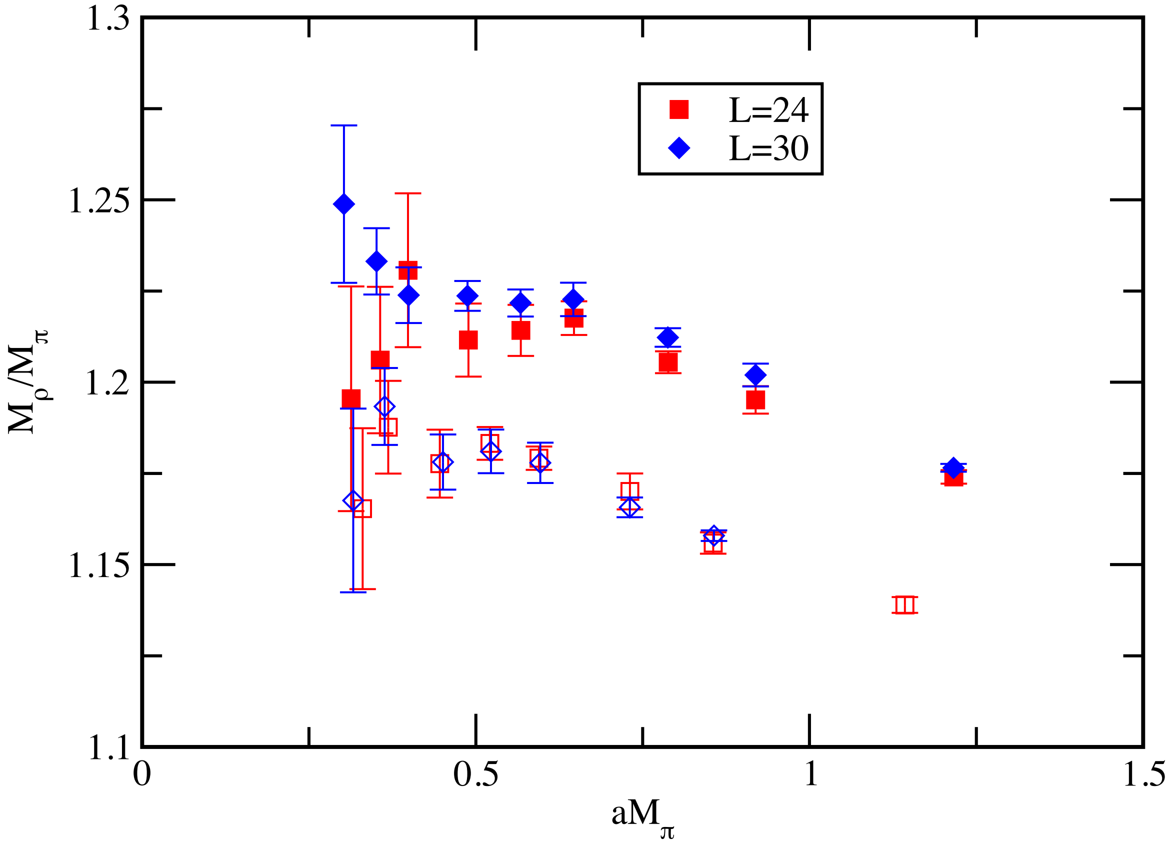


- β=3.7: small mass: consistent with hyper-scaling
- β=4.0: mass too heavy ? inconsistent with being in the hyper-scaling region

a crude analysis: M_ρ/M_π vs M_π

N_f=12: HISQ



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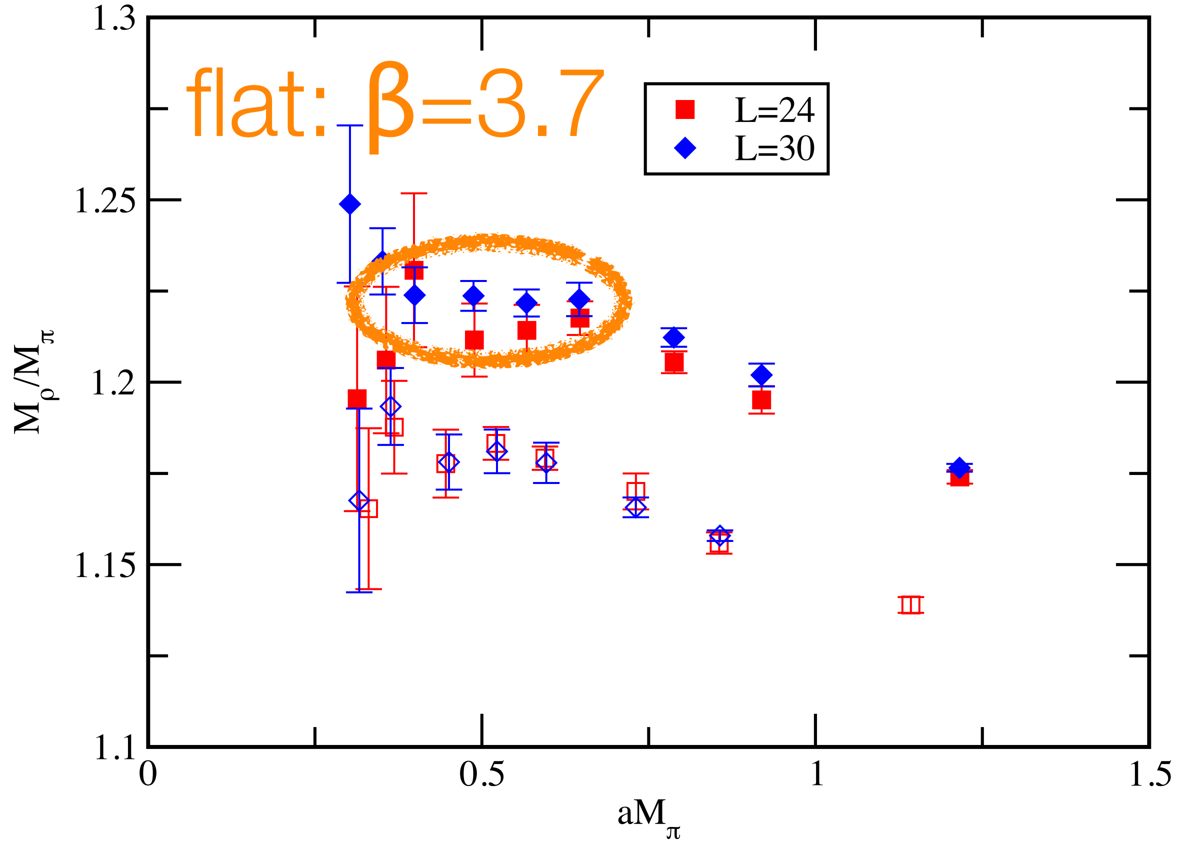

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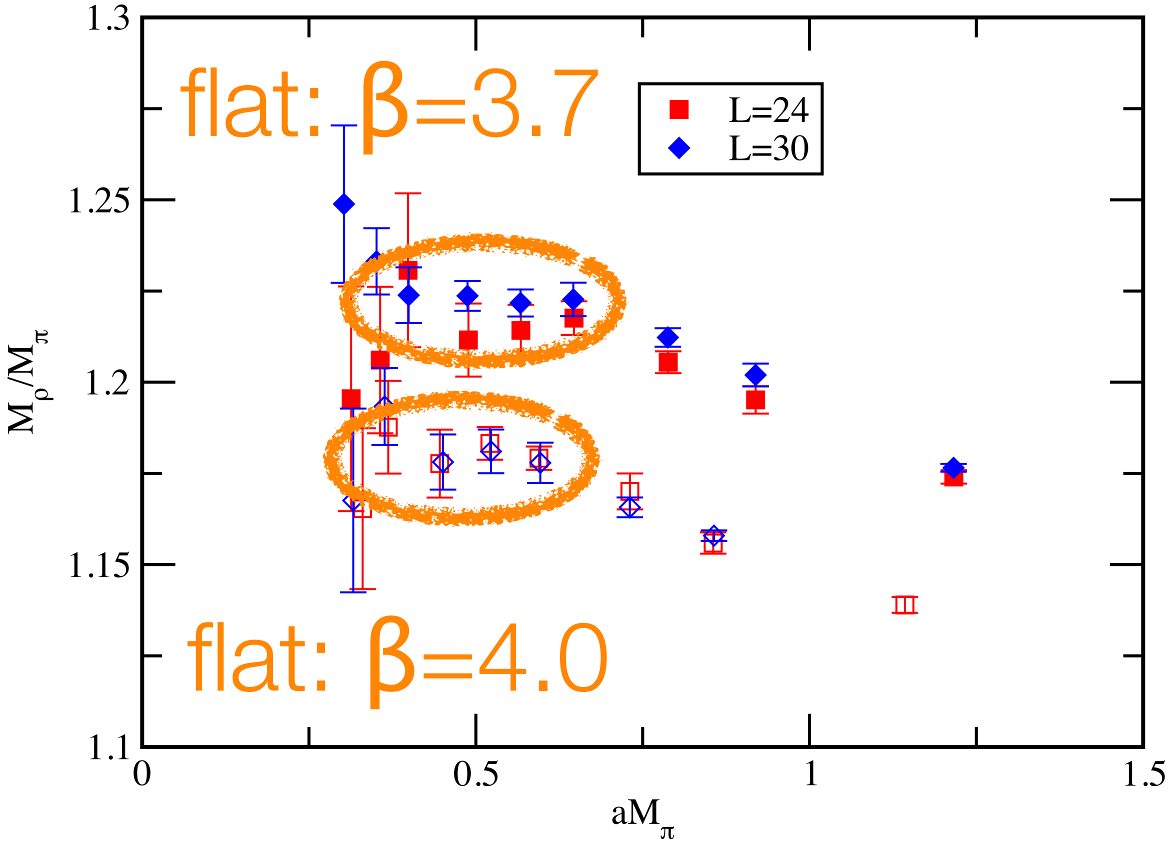

- β=3.7 & 4.0: small mass (wider than F_π): consistent with hyper scaling (HS)

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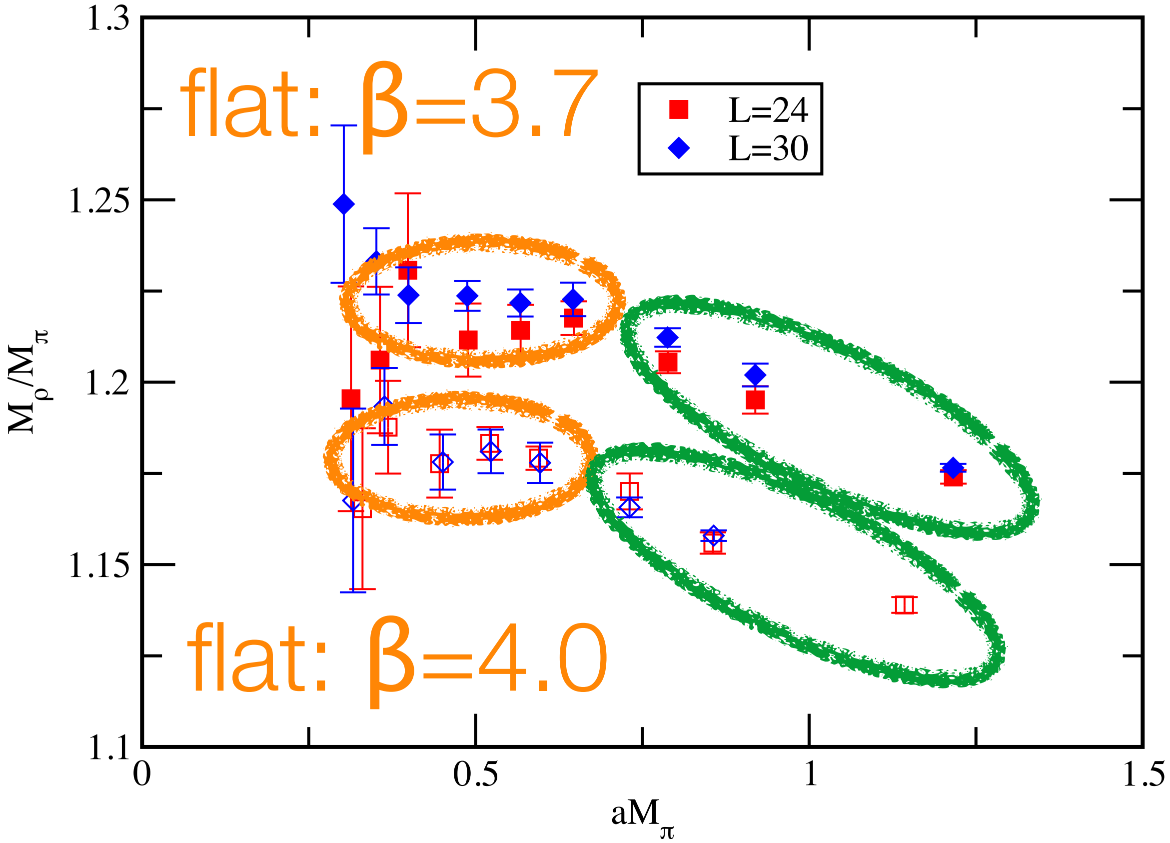

- β=3.7 & 4.0: small mass (wider than F_π): consistent with hyper scaling (HS)
- mass dependence at the tail is due to non-universal mass correction to HS

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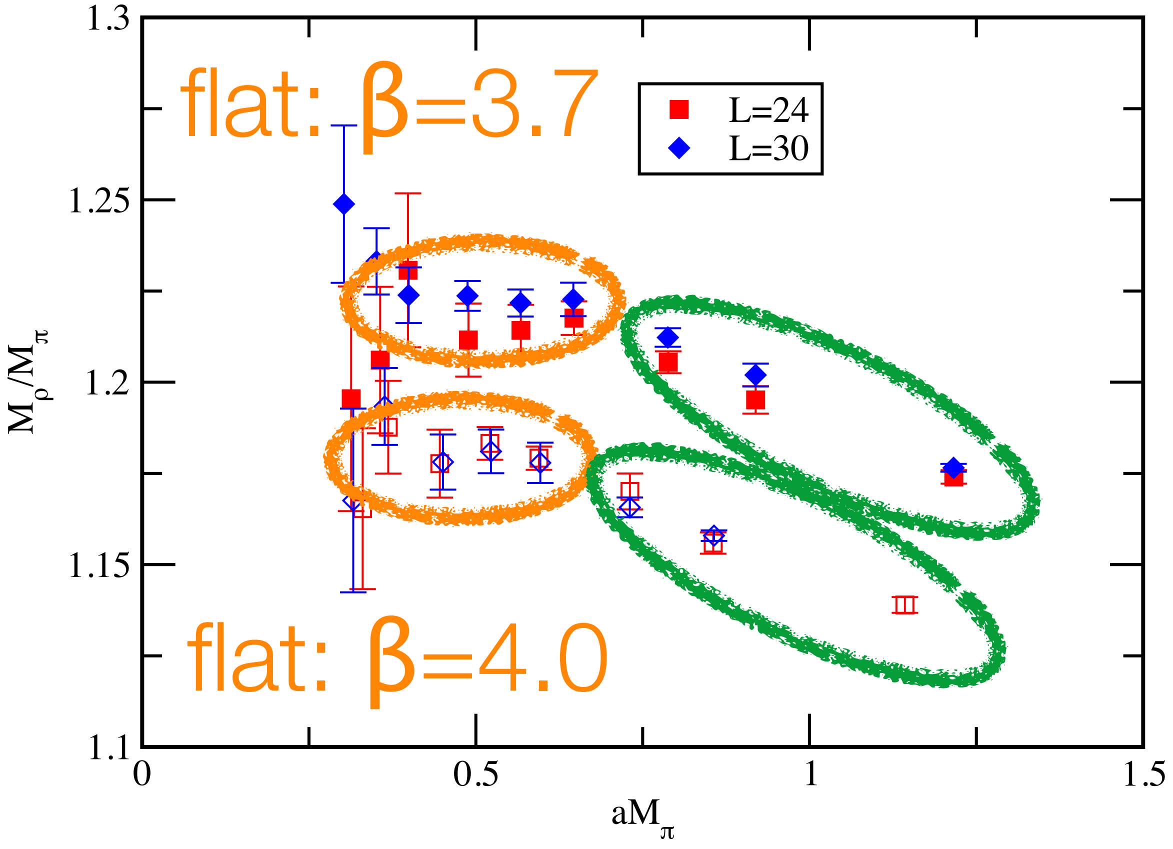

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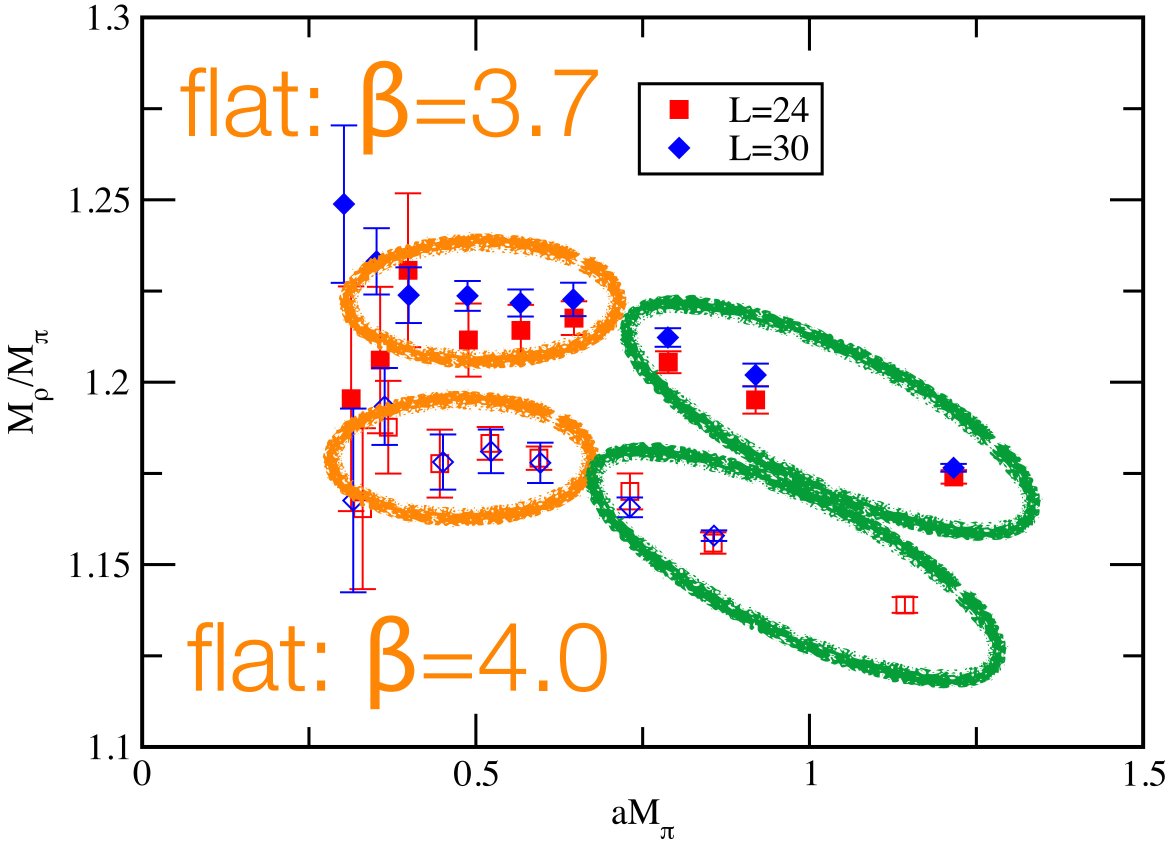

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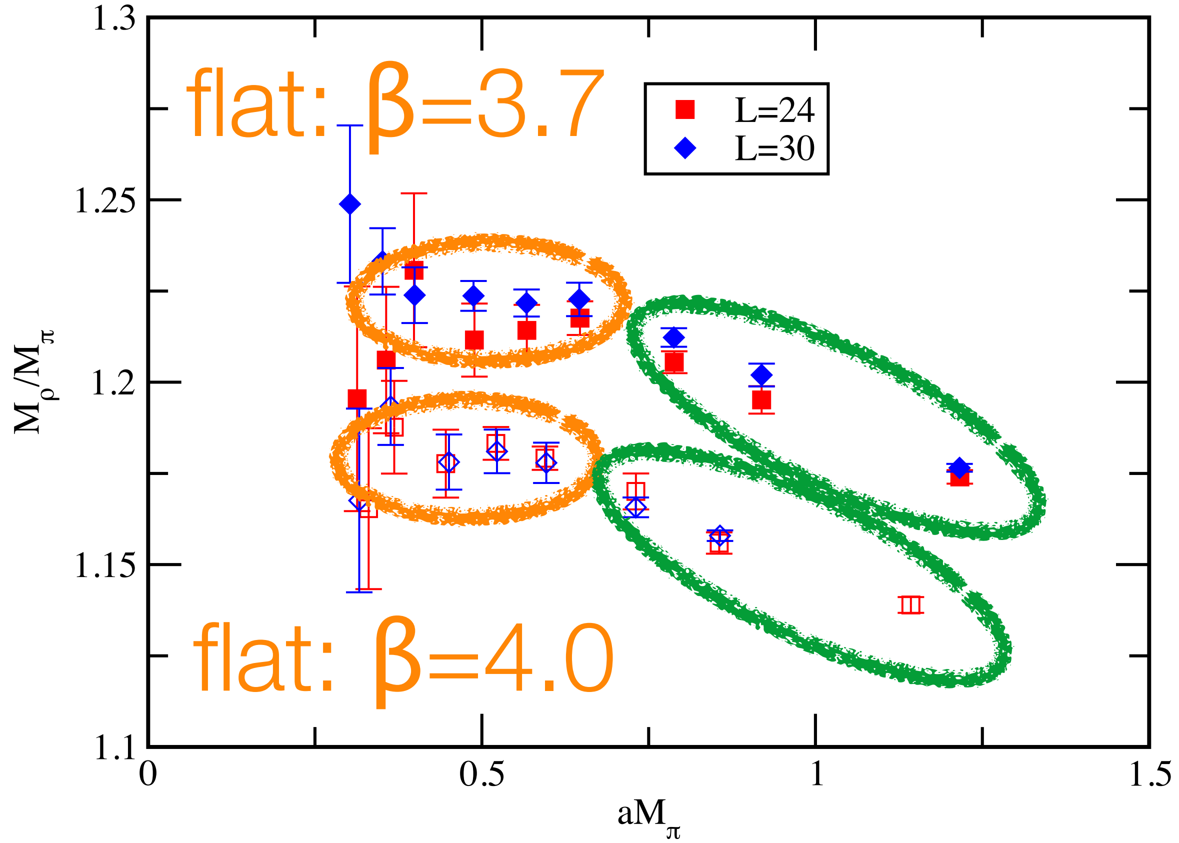


- one can attempt to perform a matching
- a(β=3.7) > a(β=4.0)
  - movement: correct direction in asymptotically free domain !

- β=3.7 & 4.0: small mass (wider than F_π): consistent with hyper scaling (HS)
- mass dependence at the tail is due to non-universal mass correction to HS

conformal (finite size) scaling

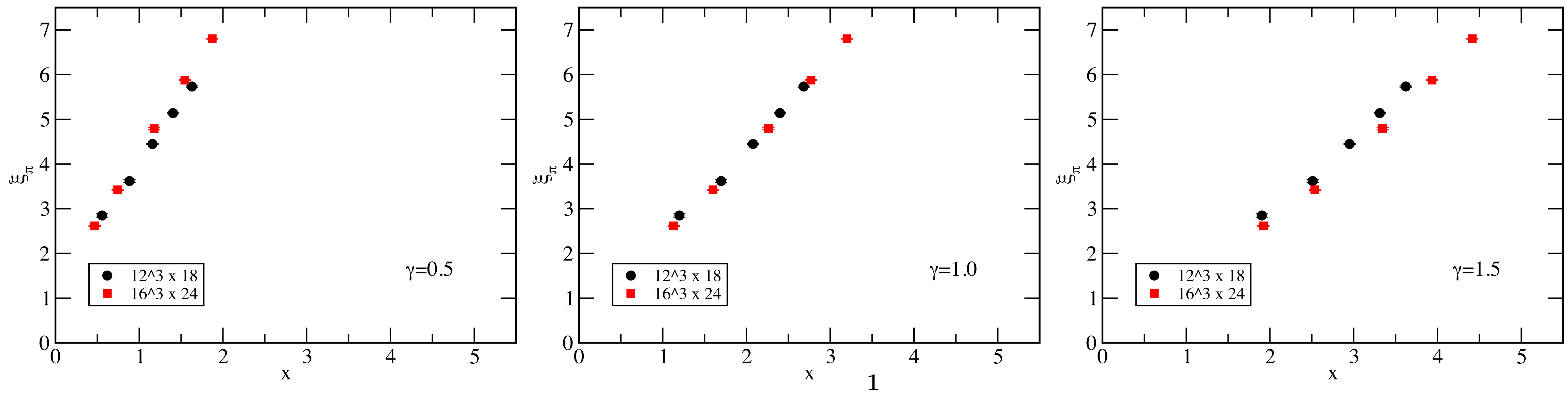
- Scaling dimension at IR fixed point [Wilson-Fisher]; Hyper Scaling [Miransky]
- mass dependence is described by anomalous dimensions at IRFP
  - quark mass anomalous dimension γ^*
  - operator anomalous dimension
- meson mass and pion decay constant obey same scaling

$$m_\pi = c_m m_f^{\frac{1}{1+\gamma^*}} \qquad f_\pi = c_f m_f^{\frac{1}{1+\gamma^*}}$$

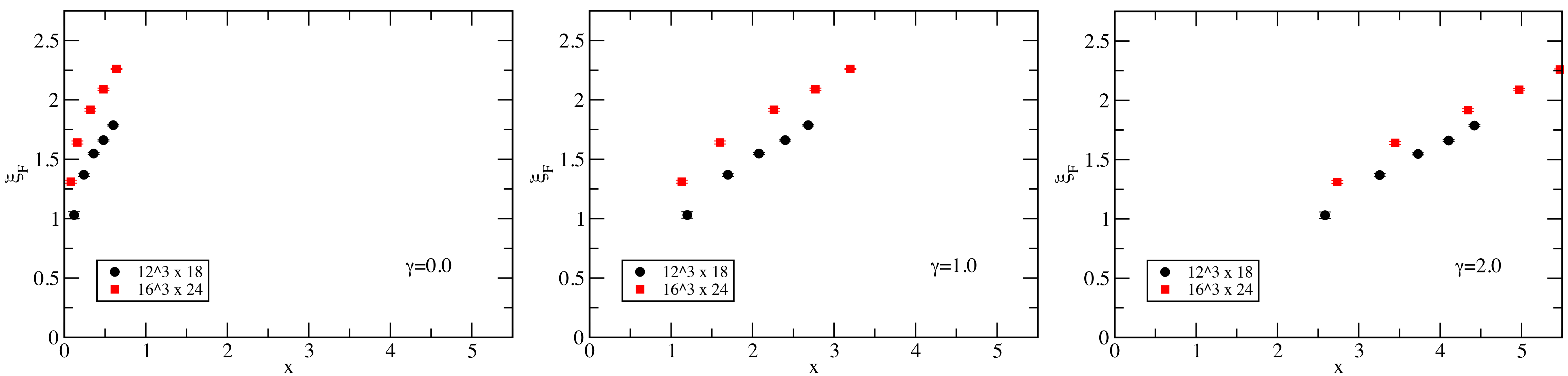
- finite size scaling in a L[4] box (DeGrand; Del Debbio et al)
  - scaling variable: $x = L m_f^{\frac{1}{1+\gamma^*}}$

$$L f_\pi = F(x) \qquad L m_\pi = G(x)$$

N_f=4 see if data align at some γ



$$x = L m_f^{\frac{1}{1+\gamma}}$$



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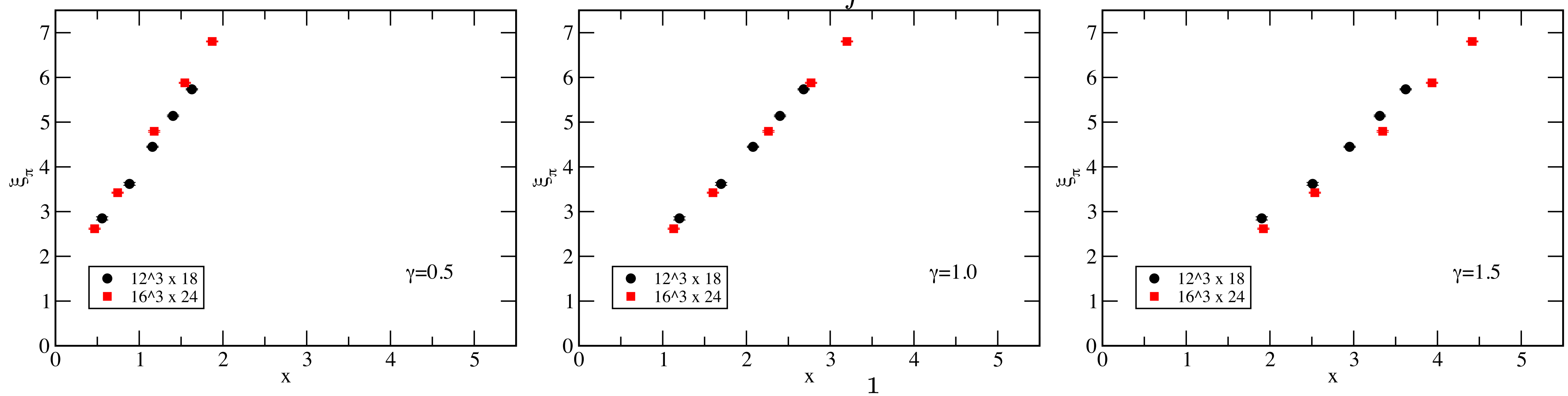
$$M_\pi L \propto m_f^{1/2} L$$



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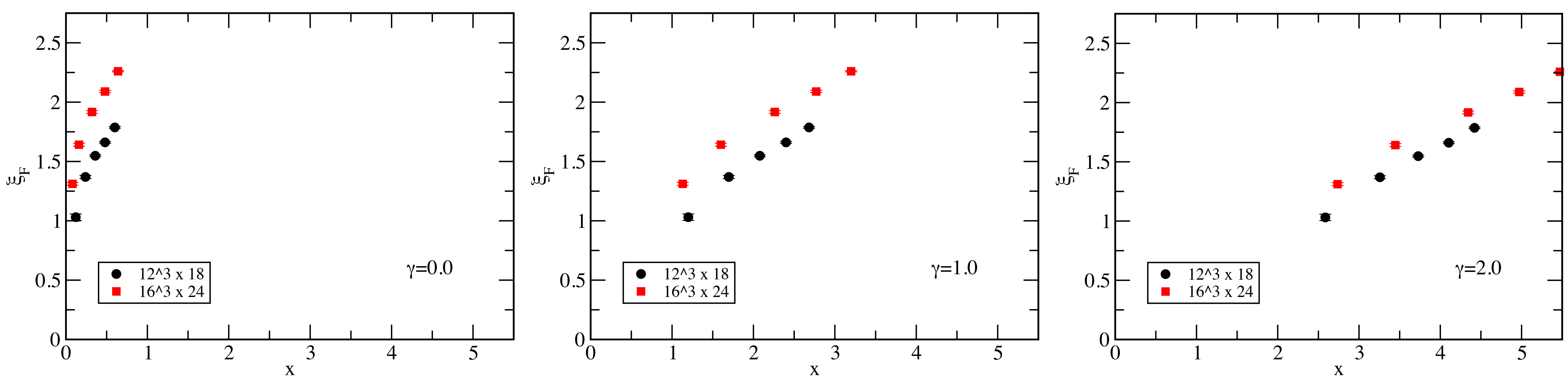

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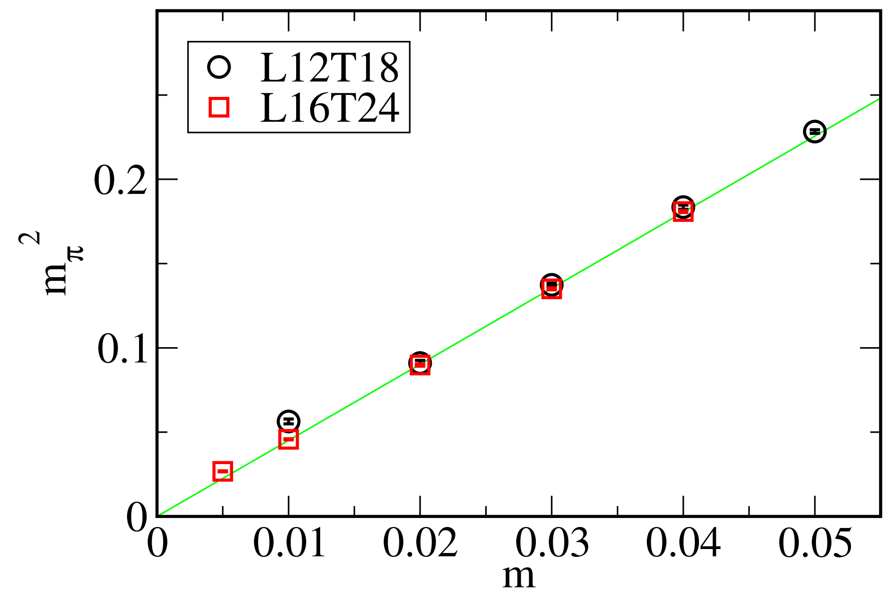

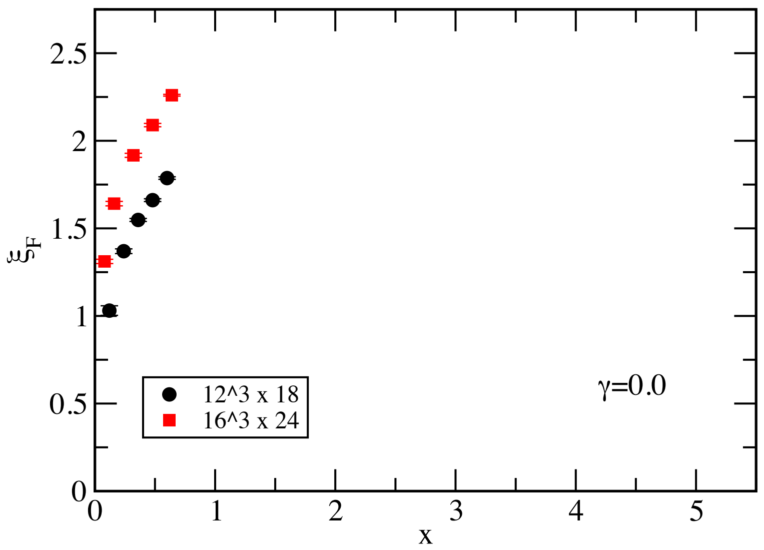

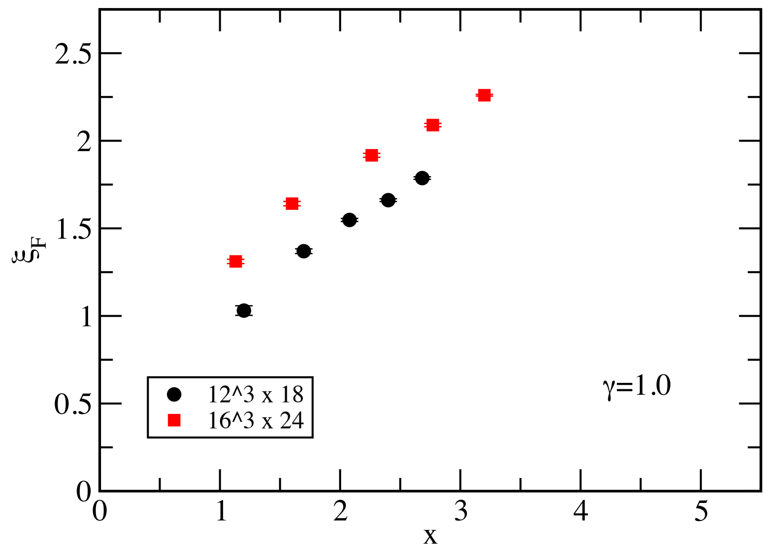

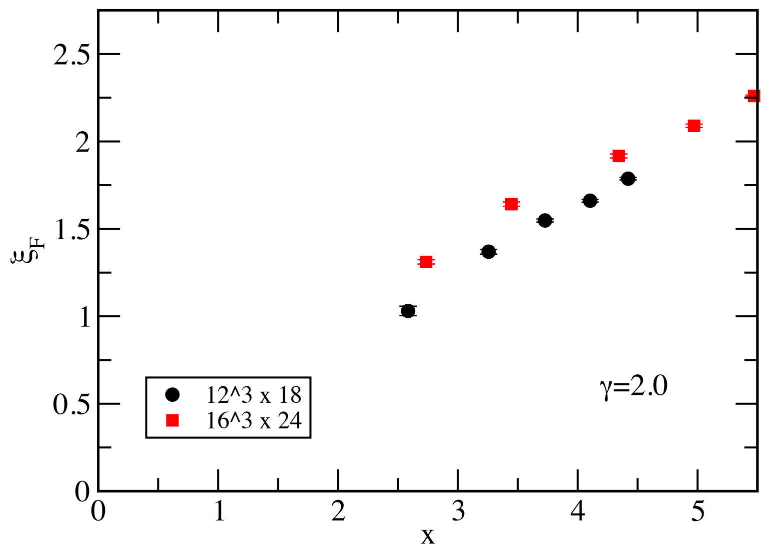

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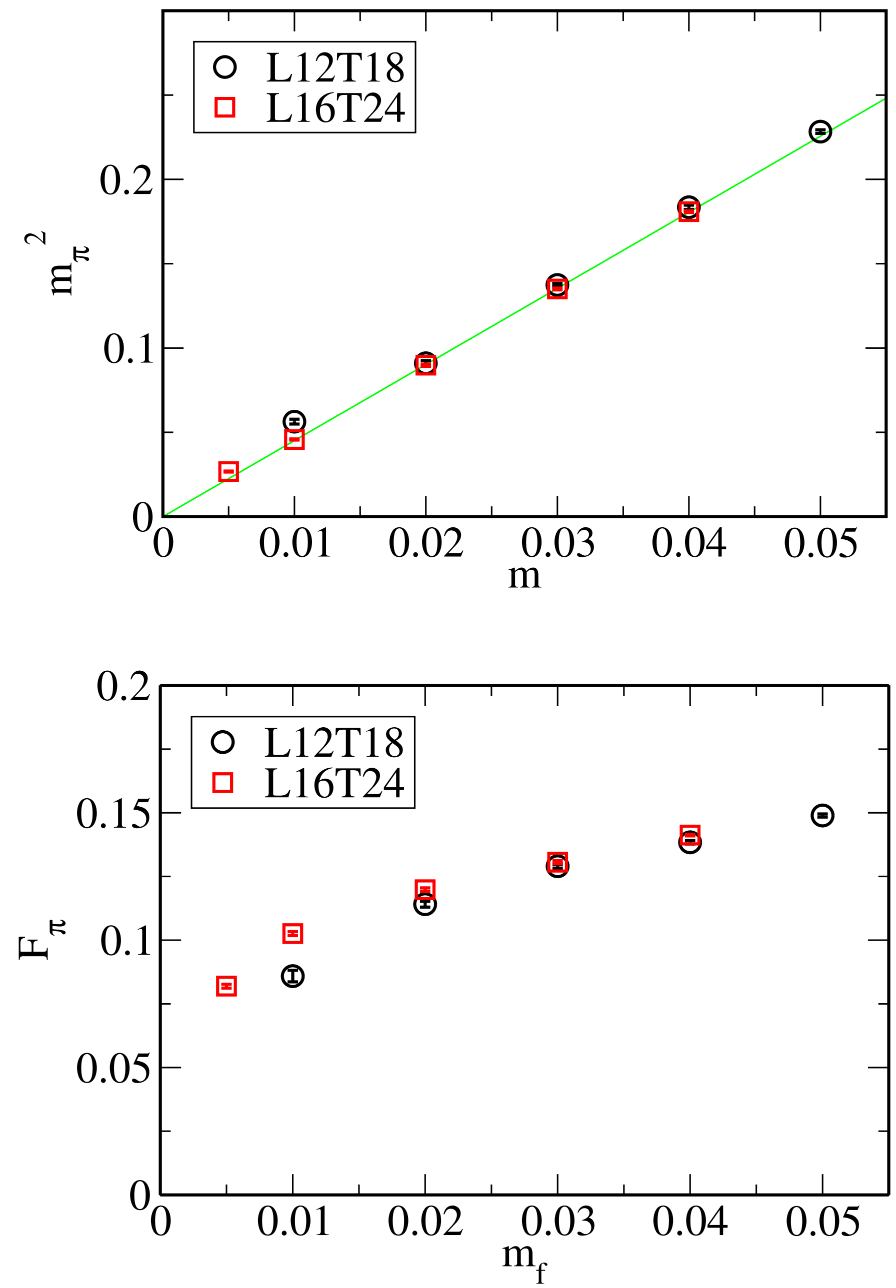
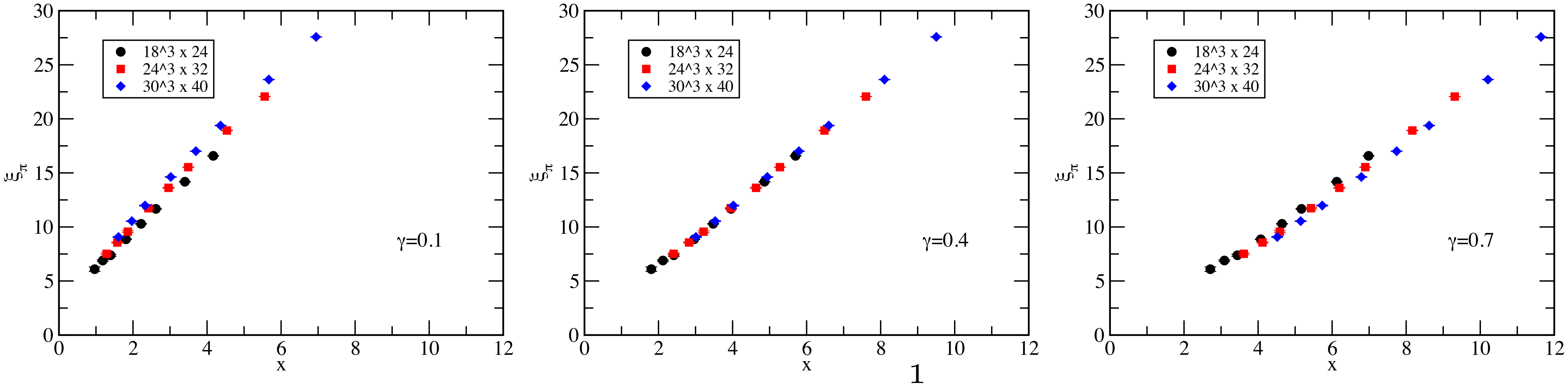
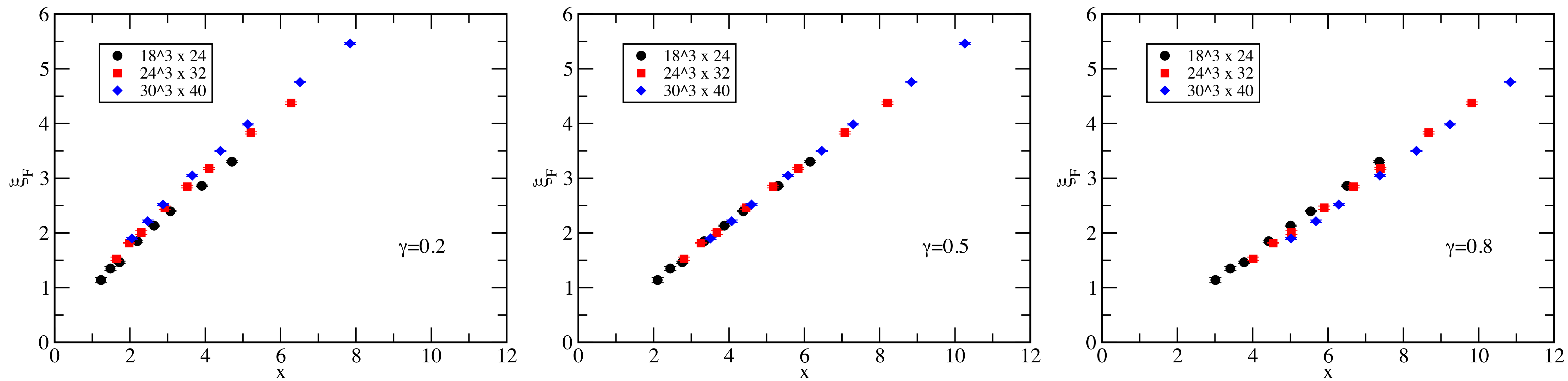


N_f=12 see if data align at some γ



$$x = Lm_f^{\frac{1}{1+\gamma}}$$



to quantify the “alignment” without resorting to a model

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- γ of optimal alignment will minimize:

$$P_p(\gamma) = \frac{1}{\mathcal{N}} \sum_{K} \sum_{j \notin K} \frac{|\xi_p^j - f_p^{(K)}(x_j)|^2}{\delta^2 \xi_p^j}$$

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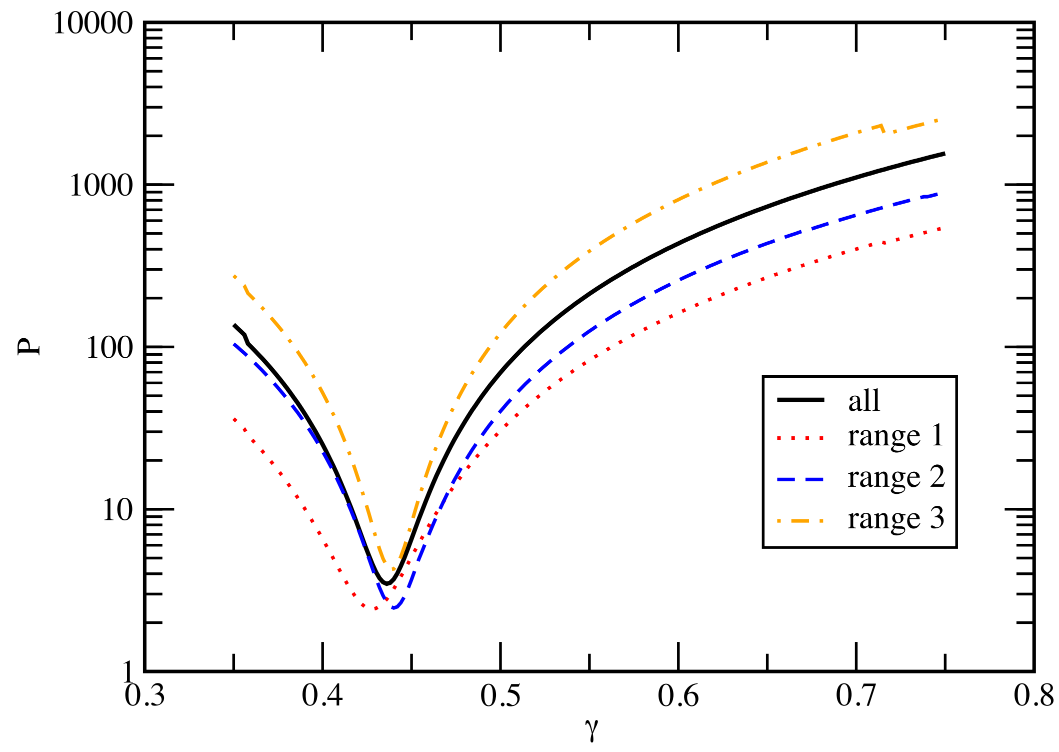

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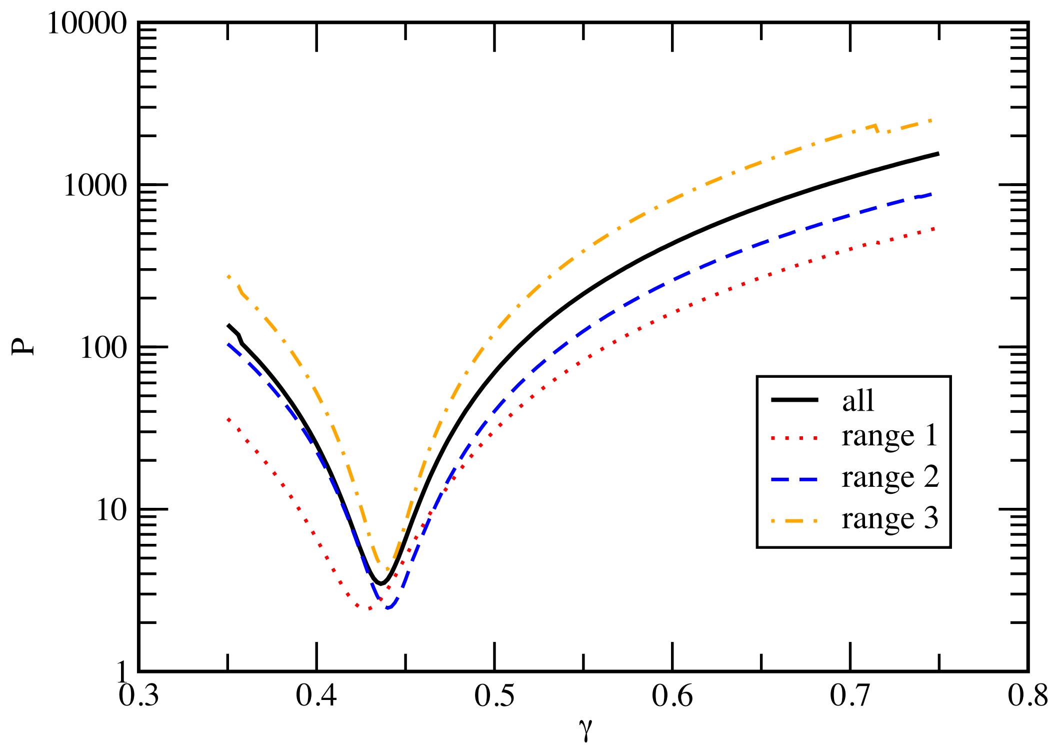

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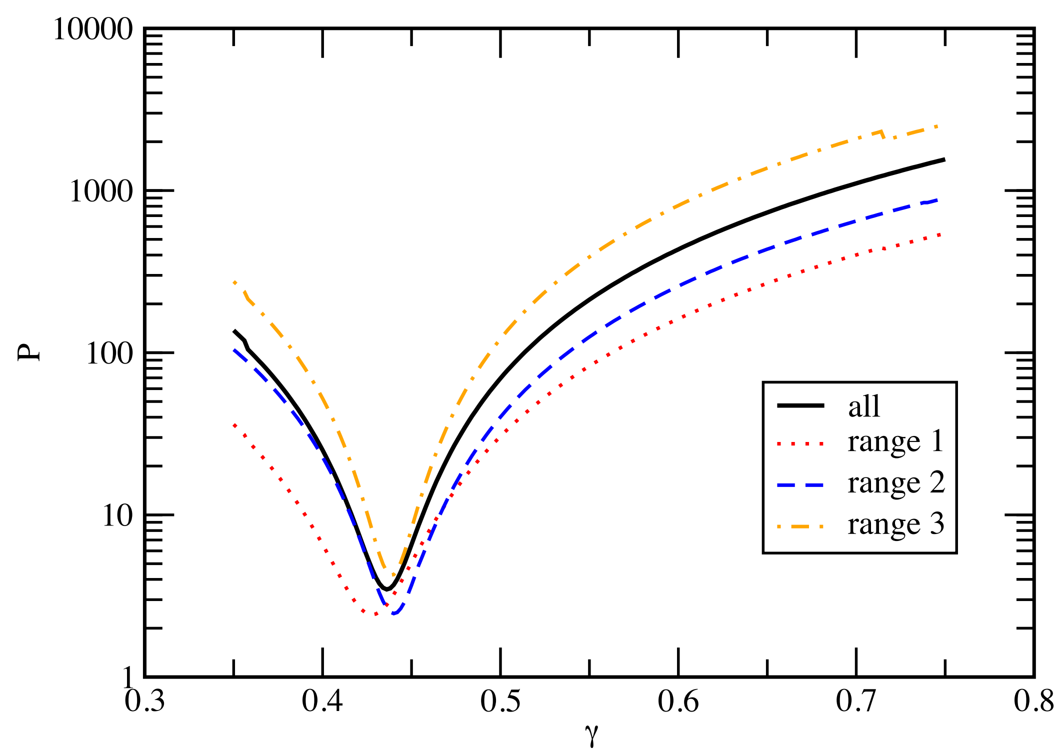
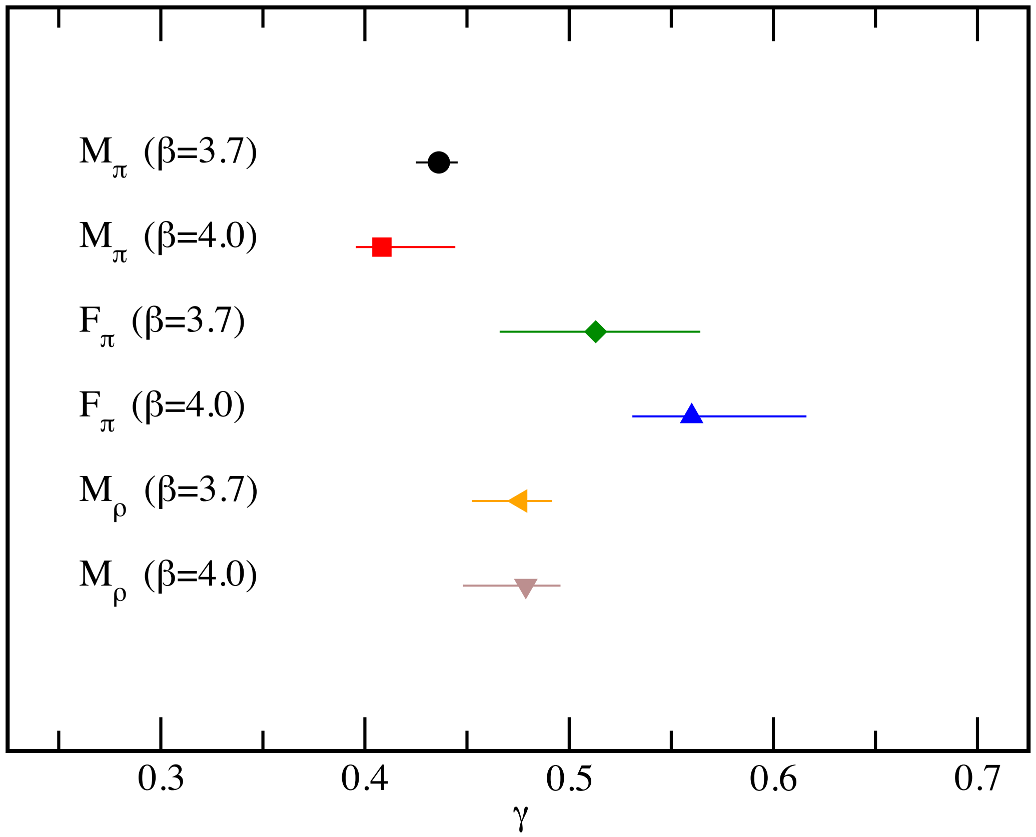


- systematic error due to small L, large m estimated by examining the x and L range dependence

summary of γ obtained by minimizing P(γ)



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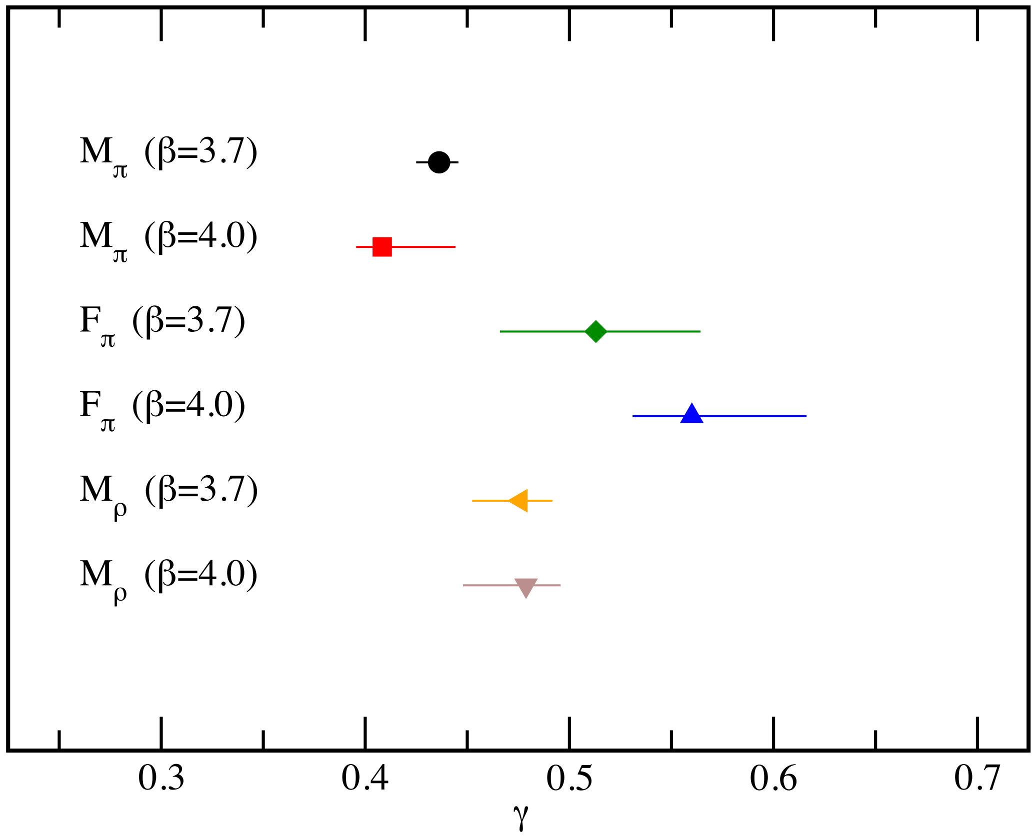

- consistent γ by 1.5 σ level except for F_π at β=4.0

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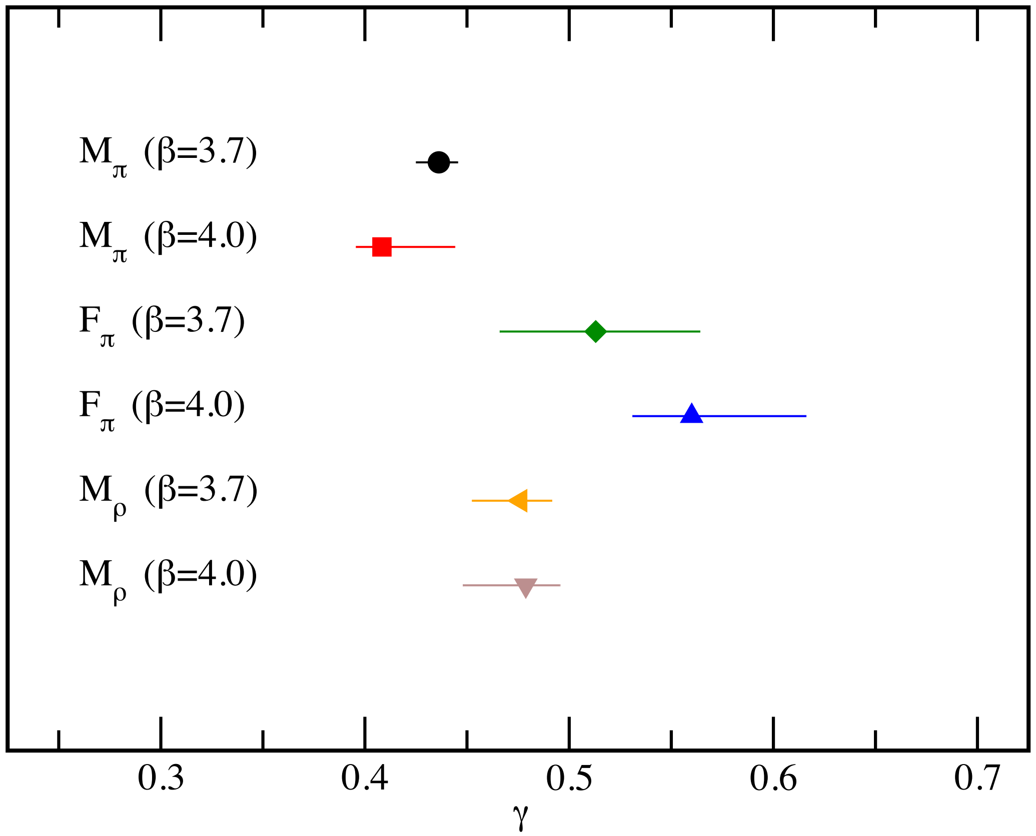

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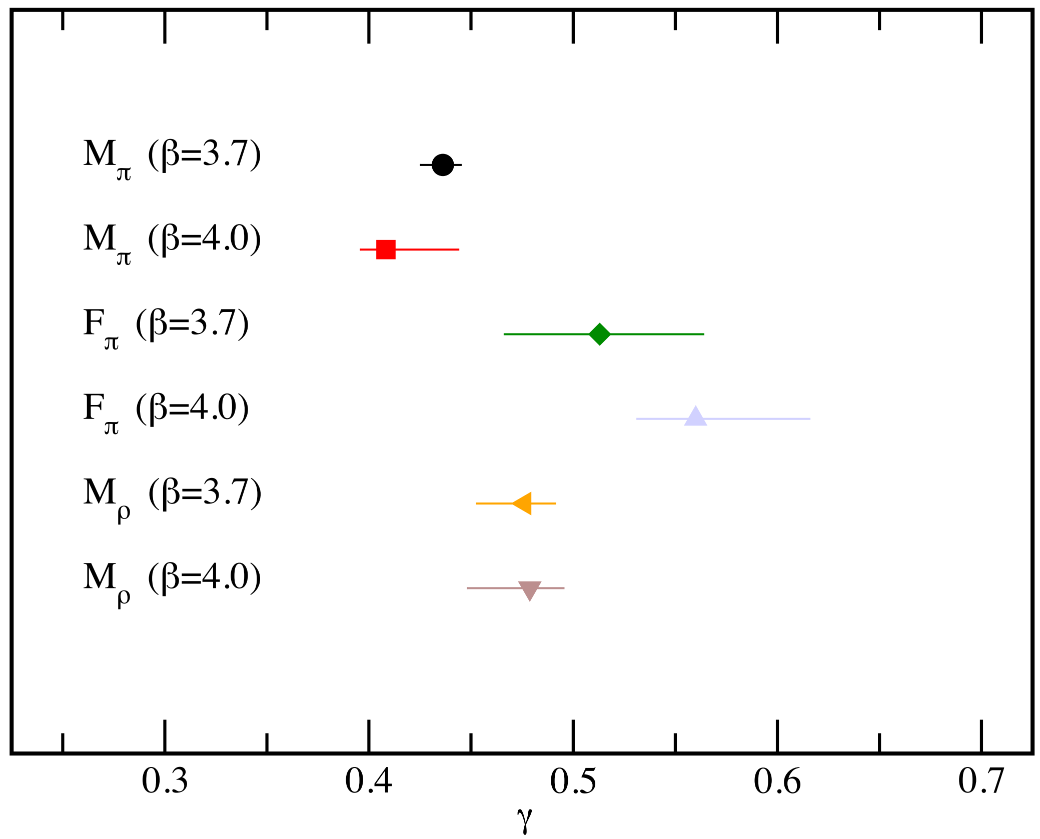


- consistent γ by 1.5 σ level except for F_π at β=4.0
- remember: F_π at β=4.0 speculated to be out of the scaling region
- universal low energy behavior: good with $0.4<\gamma_*<0.5$

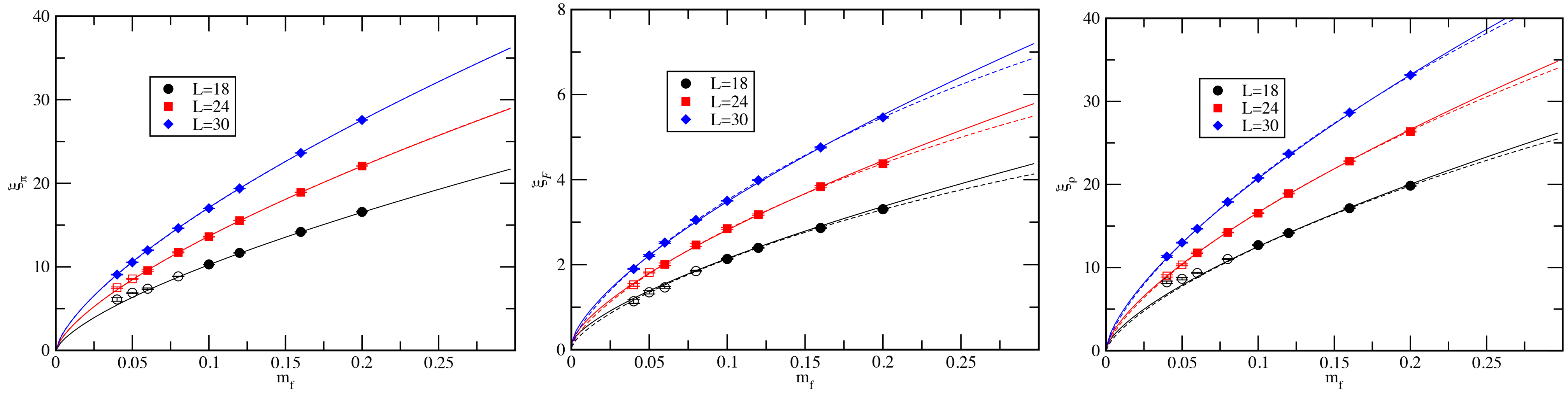
Conformal type fit with finite volume correction

$$\xi = LM_\pi,\ LF_\pi,\ LM_\rho$$

$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)} \quad \cdots \text{fit a},$$

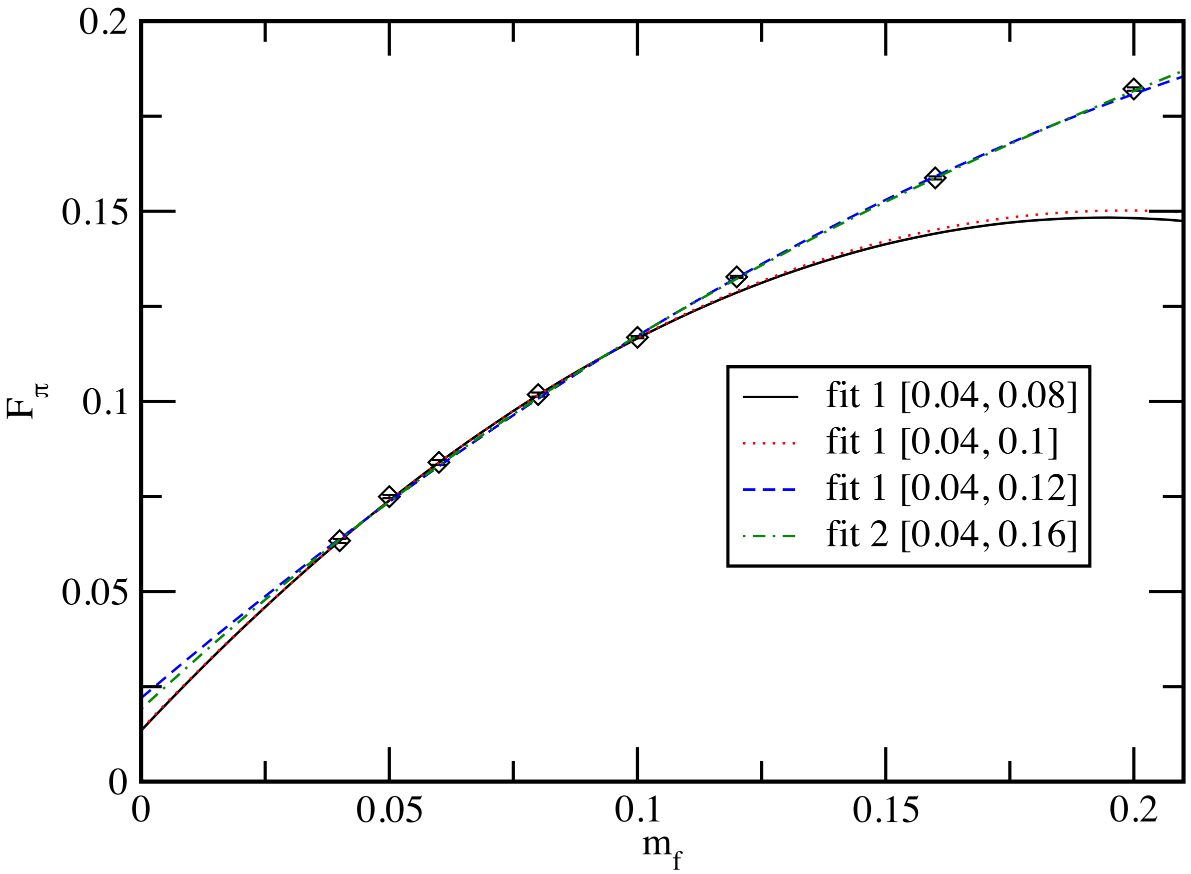
$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)} + c_2 L m_f^{\alpha} \quad \cdots \text{fit b}.$$

	γ	α	χ^2/dof
fit a	0.455(3)	-	5.43
fit b-1	0.417(10)	$\frac{(3-2\gamma)}{(1+\gamma)}$	1.88
fit b-2	0.431(8)	[2]	1.83



- simultaneous fit it with a leading mass dependent correction is not bad
  - b-1: Ladder Schwinger-Dyson, b-2: $(am)^2$ lattice artifact
- resulting γ is consistent with the model independent analysis

ChPT fit (after infinite volume extrapolation)



$$h(m_f) = \begin{cases} c_0 + c_1 m_f + c_2 m_f^2 & \cdots \text{fit1} \\ c_0 + c_1 m_f + c_2 m_f^2 + c_3 m_f^3 & \cdots \text{fit2} \end{cases}$$

fit range	c_0	c_1	c_2	c_3	χ^2/dof
fit 1 : [0.04, 0.08]	0.0101(54)	1.53(19)	-4.8(1.5)	-	2.09
fit 1 : [0.04, 0.1]	0.0138(29)	1.39(88)	-3.62(61)	-	1.39
fit 1 : [0.04, 0.12]	0.0226(17)	1.113(45)	-1.64(27)	-	5.42
fit 2 : [0.04, 0.16]	0.0182(34)	1.28(13)	-3.4(1.3)	6.0(4.2)	4.75

- 2nd order polynomial fit is reasonably good for small mass range & c_0>0

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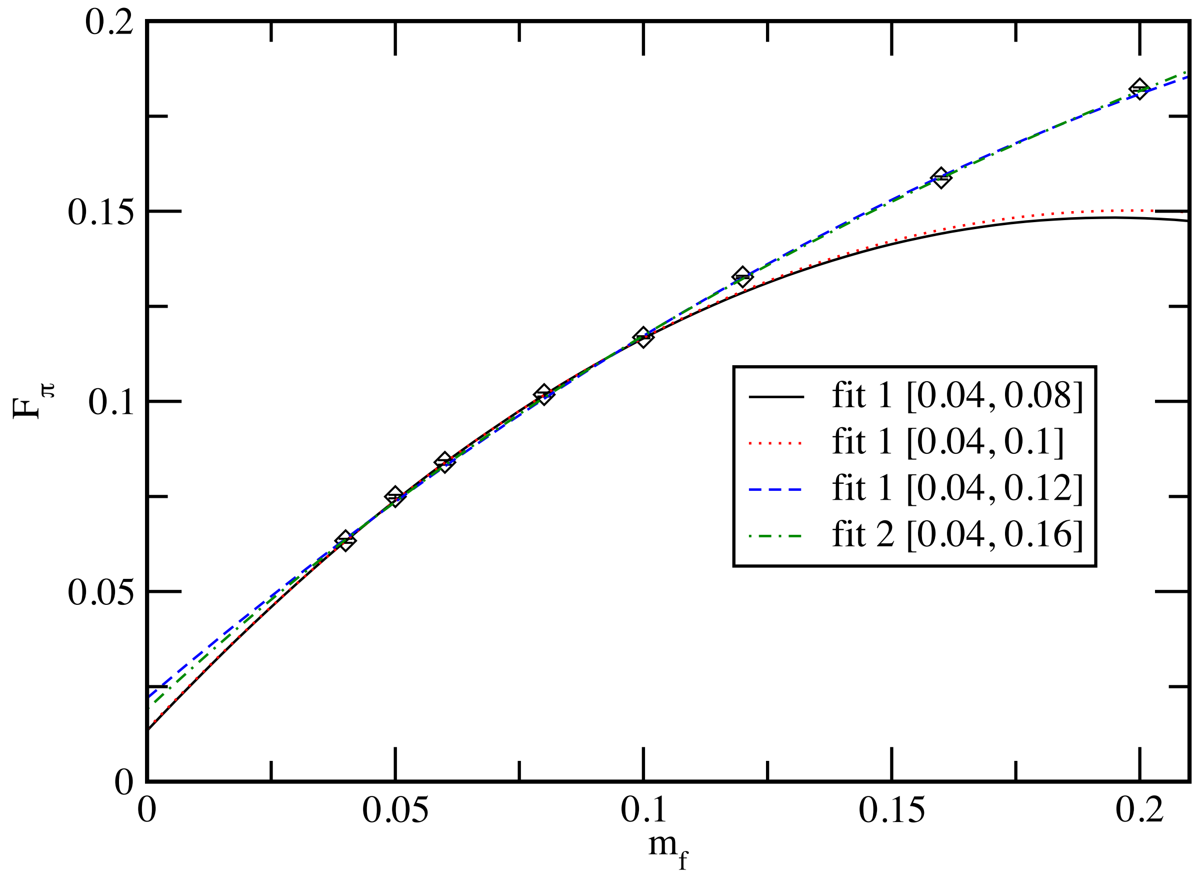

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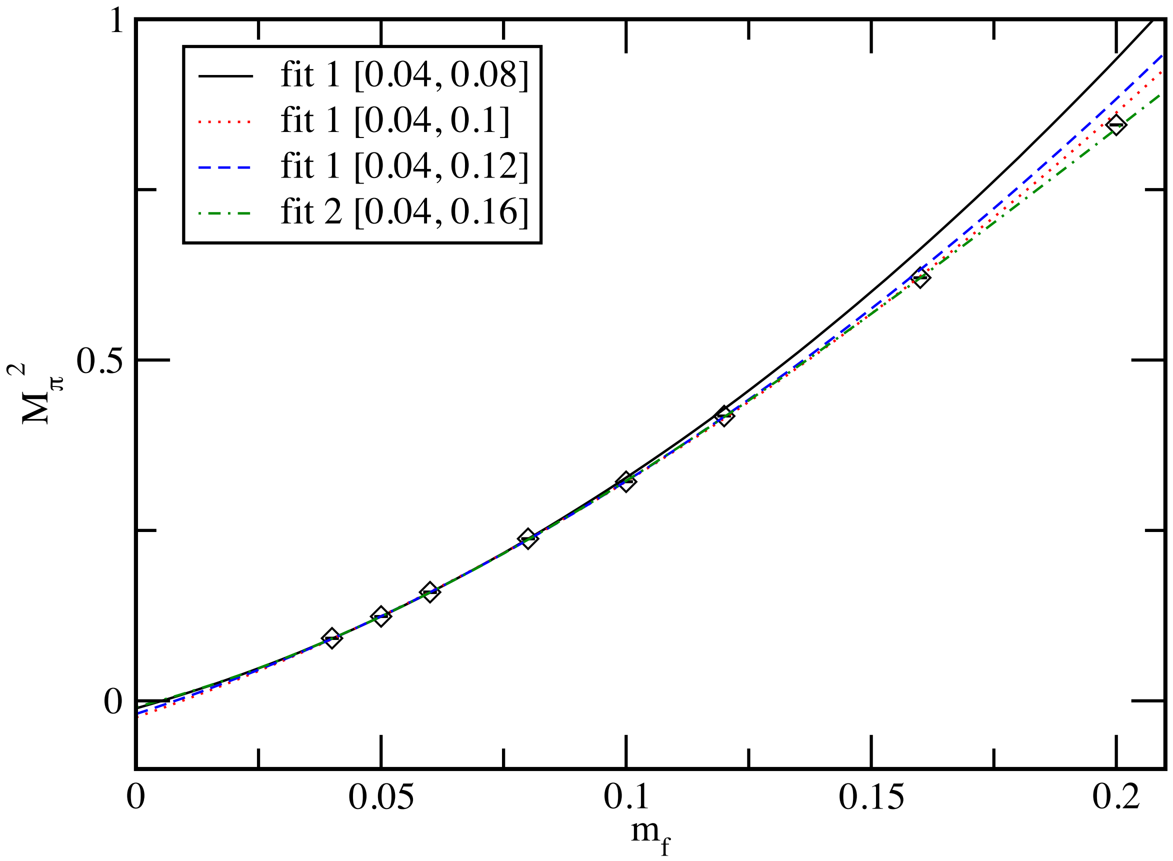

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fit 1 : $[0.04, 0.08]$	-0.0090(93)	1.95(32)	14.2(2.6)	-	0.16
	[0]	1.640(31)	16.68(47)	-	0.56
fit 1 : $[0.04, 0.1]$	-0.0232(50)	2.46(16)	9.9(1.1)	-	1.75
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fit 2 : $[0.04, 0.16]$	-0.0044(61)	1.69(22)	19.1(2.4)	-32.9(7.6)	3.28
	[0]	1.537(29)	20.76(53)	-38.2(2.3)	2.59

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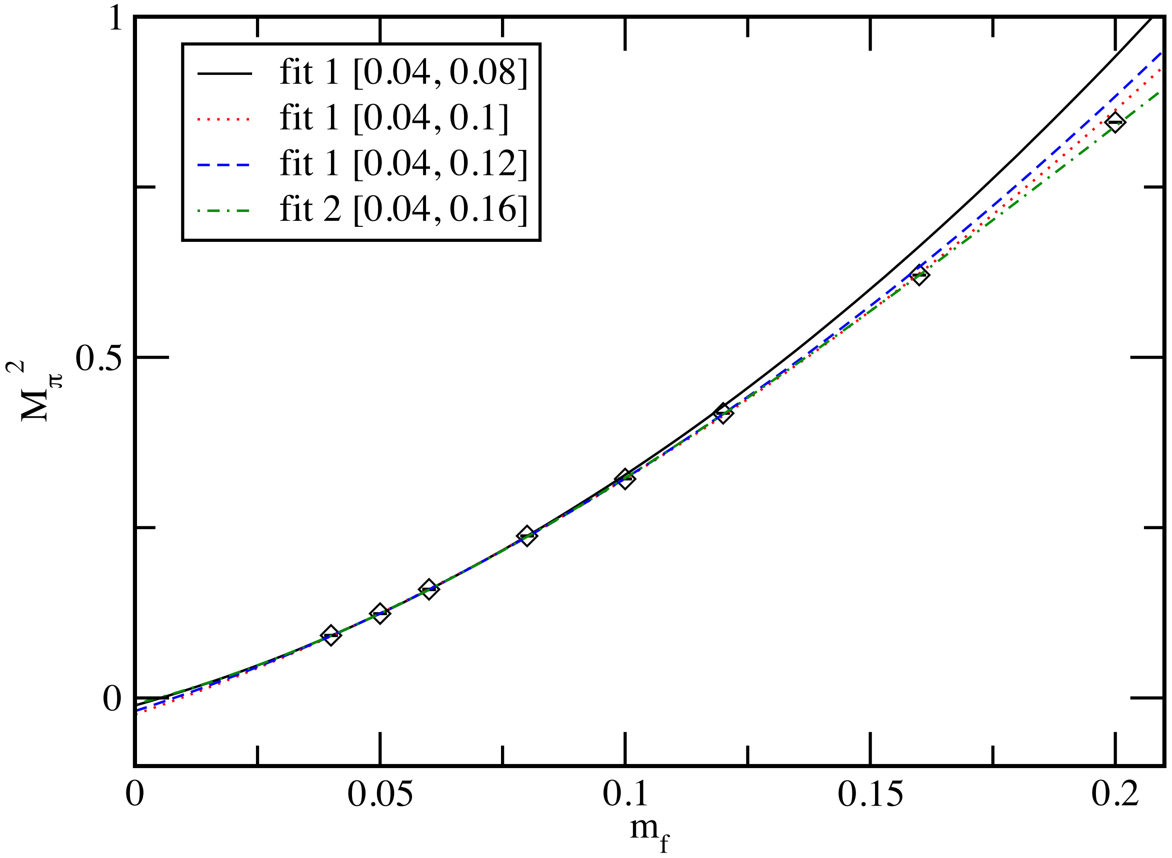

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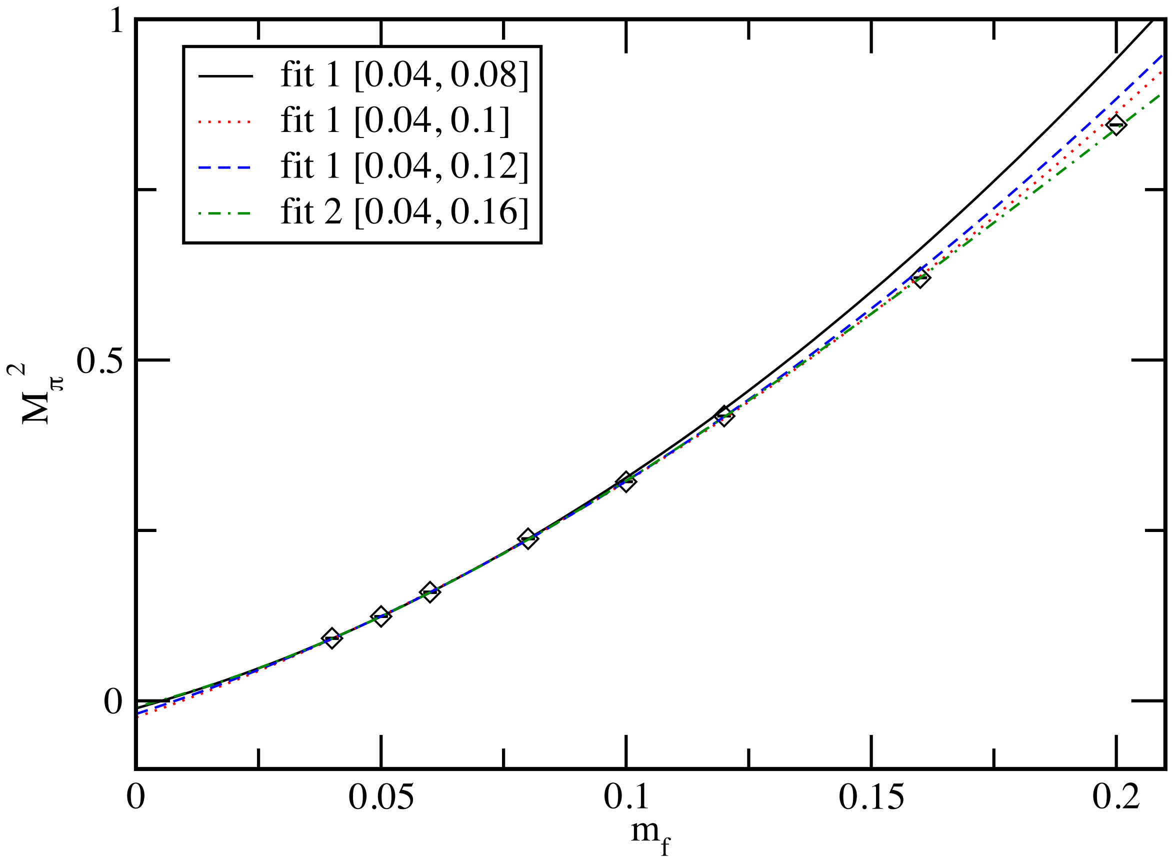

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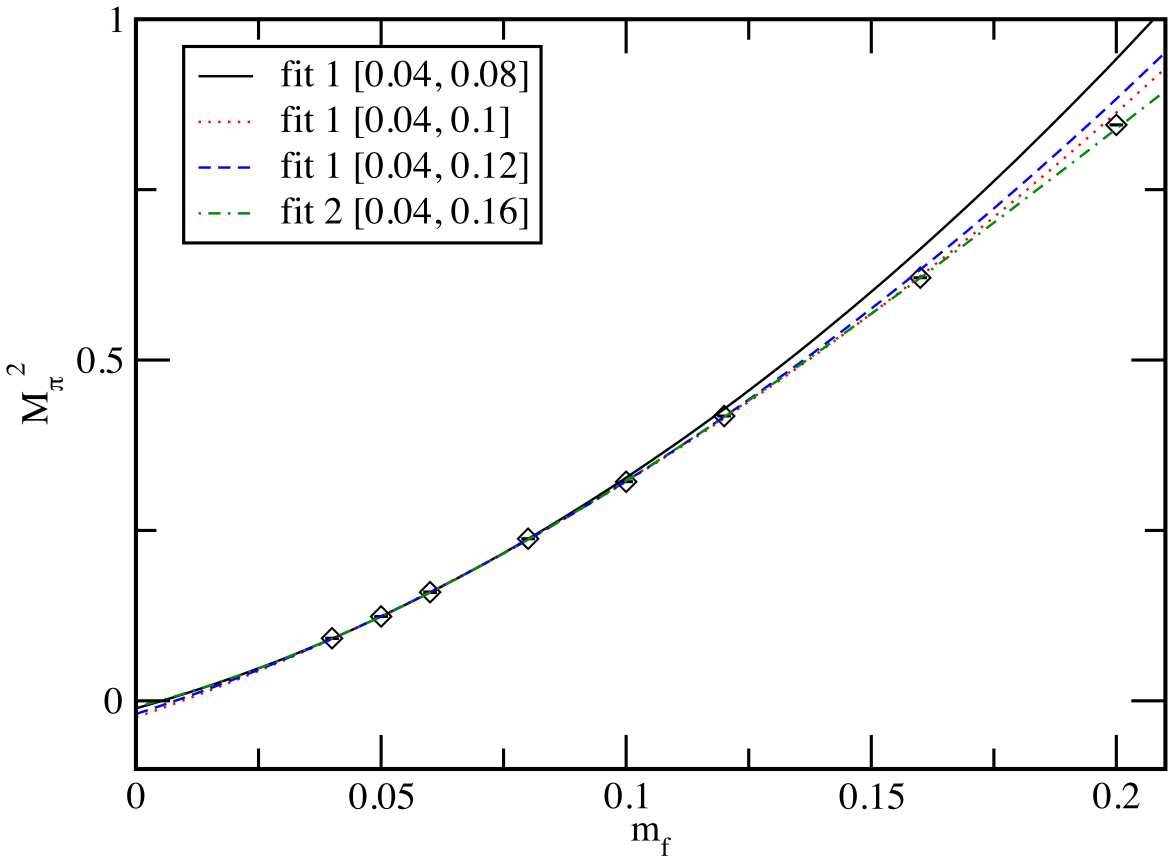

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- But: $M_\pi/(4\pi F)\sim 2$ at lightest point → difficult to tell real chiral behavior

Summary:

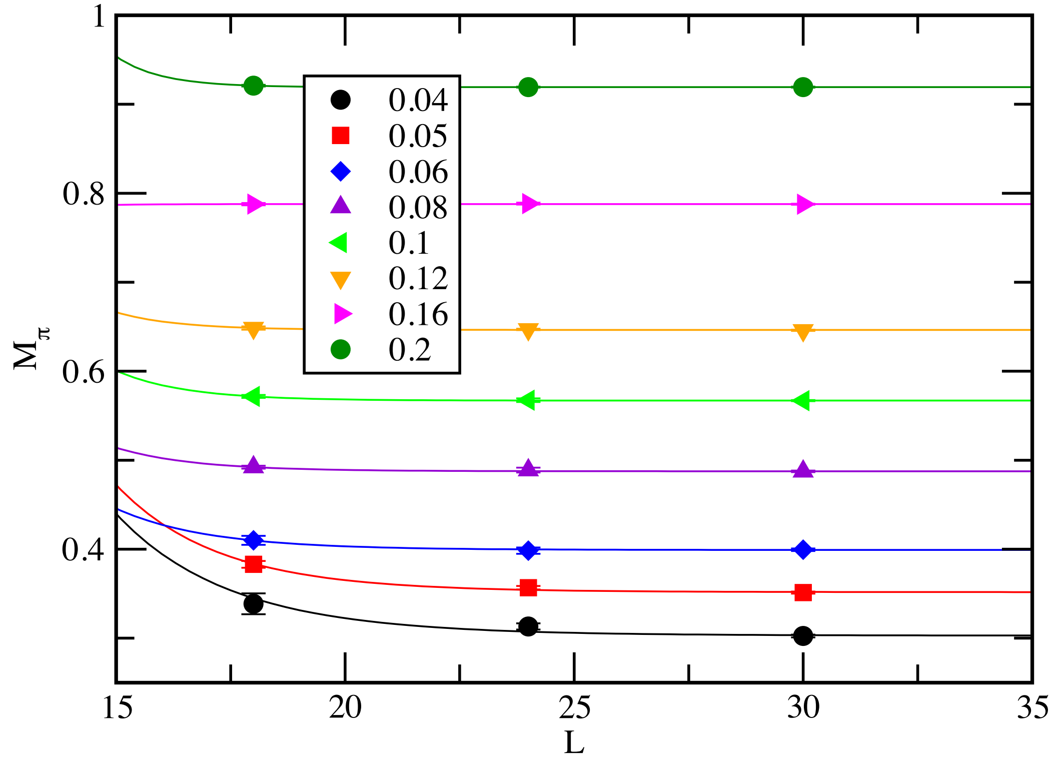
SU(3) gauge theory with N_f=12 fundamental fermion simulation with HISQ

- β=3.7, 4.0: consistent with being in the asymptotically free regime
- M_π, F_π, M_ρ: consistent with the finite size hyper scaling for conformal theory
- resulting γ^* from different quantities, lattice spacings are consistent except
  - F_π at β=4.0 (m_f likely too heavy for universal mass dep. to dominate)
- need careful continuum scaling needed to get more accurate than $0.4<\gamma^*<0.5$
- real / remnant (approximate) conformal property is definitely there
- could not exclude SχSB with very small breaking scale
- even if SχSB, γ_m too small for walking theory of phenomenological interest
- N_f=8 theory is interesting & under investigation with same lattice set up

Thank you for your attention

ChPT inspired infinite volume limit (β=3.7)

$$M_\pi(L) - M_\pi = c_{M_\pi} \frac{e^{-LM_\pi}}{(LM_\pi)^{3/2}}$$

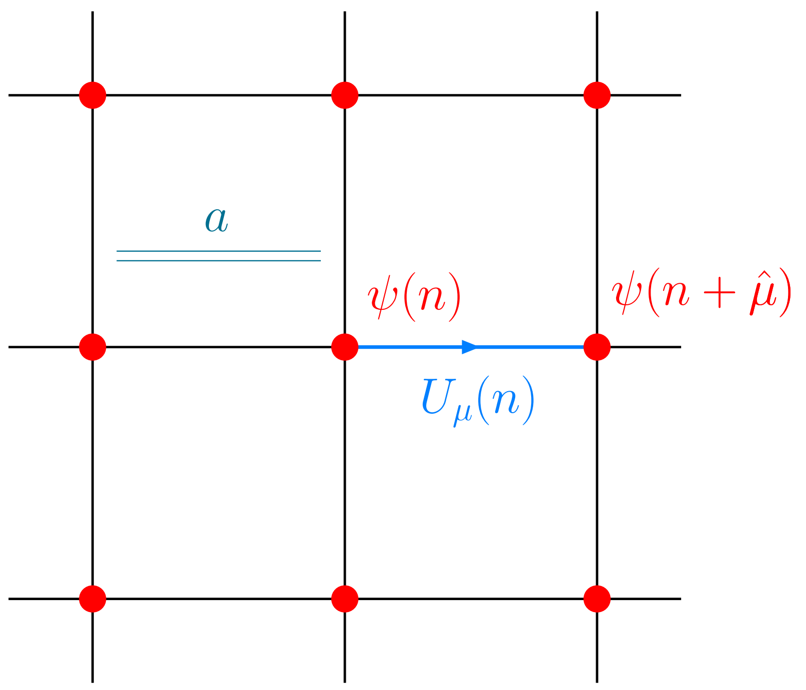


- ChPT type finite volume effect → chiral fit results not inconsistent with S χ SB

HISQ action

- proposed by HPQCD collaboration for
  - smaller taste violation than other approaches
  - better handling of heavy quarks
- being used in simulations
  - MILC: Nf=2+1+1 QCD
  - HOTQCD: QCD thermodynamics: Bazavov-Petreczky (Lat'10 proceedings)
    - HISQ/tree is **best** of [HISQ/tree, Asqtad, stout]

      for flavor (taste) symmetry, dispersion relation



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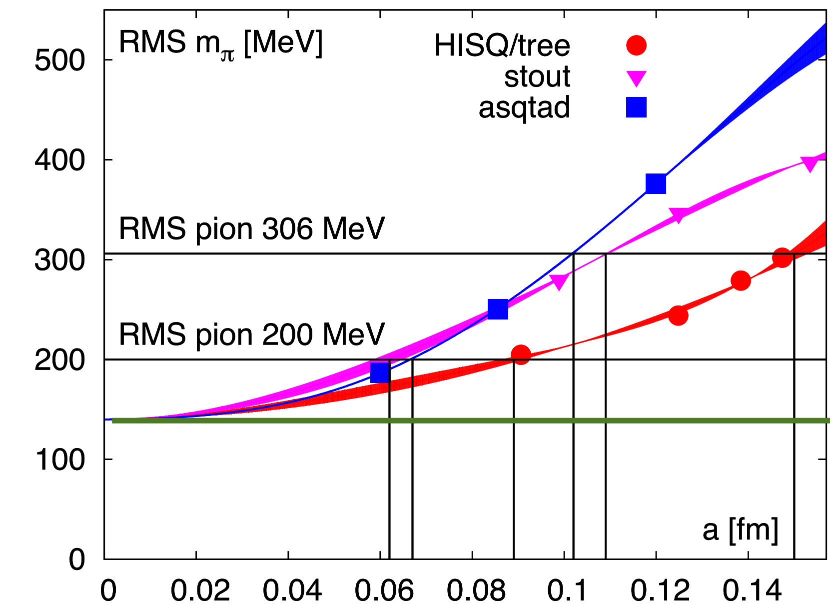

Figure 2: RMS pion mass when $m_{\gamma_5} = 140$ MeV. See details in the text.

LHC (Large Hadron Collider)

- excess @ ~125 GeV
    - 1 σ level (look elsewhere)
    - larger when ATLAS & CMS results are combined ?
    - $M_W = M_Z \cos\theta_W = gF_\pi/2$ ($F_\pi = v_{weak} = 246$ GeV)
    - M_H~500 GeV: problem ?
- even if scalar is fund at ~125 GeV
    - possible techni-dilaton (Matsuzaki-Yamawaki,,)
    - 0++ glueball tends to be much lighter than techni-hadrons
        - Cf. SU(2) lattice work by Del Debbio et al
- important to investigate glueball for SU(3) as well !!!